

科目：應用數學【物理系三年級】

1. (20%) Please prove the following Laplace transforms:

(a) $\mathcal{L}(\sin \omega t) = \frac{\omega}{s^2 + \omega^2}$;

(b) $\mathcal{L}(f'') = s^2 F(s) - sf(0) - f'(0)$.

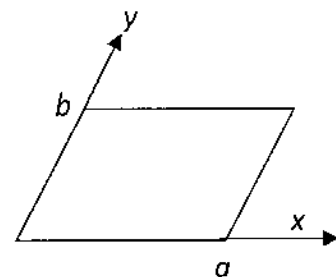
2. (20%) Find the Fourier-series of a rectangular wave

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ 2k & 0 < x < \pi \end{cases} \text{ and } f(x + 2\pi) = f(x)$$

3. (20%) Please calculate (with details)

$$\oint_C \frac{\tan z}{z^2 - 1} dz \text{ with } C: |z| = 3/2$$

4. (40%) A 2 dimensional thermal conducting sheet of lengths a and b along x and y axes. The temperature distribution of the edge at $x=0$ equals to $f(y)$. The temperature of the edge at $x = a$ equals to a constant T_0 all the time. The temperature at both edge of $y = 0$ and $y = b$ equal to zero. When the thermal conduction equation is $\nabla^2 T(x, y, t) = \frac{1}{c^2} \frac{\partial T(x, y, t)}{\partial t}$



where c is a constant.

(a) (20%) Please use the method of the separation of variables to separate the time dependent partial differential equation into ordinary differential equations.

(b) (20%) Please find the final distribution of temperature of the rectangular sheet as a function of time and coordinates.

科目：電磁學【物理系三年級】

注意：第一題(多選題)與第二題為單選題(各佔 15%)，作答時必須說明理由才全部給分。

1. 一電流源 I 加於半徑為 R 的圓形平行板電容器兩端，如圖一所示，且兩平行板間的距離為 s (假設 $s \ll R$)。當電容器在充電的過程中，請問下列敘述何者為正確的？

(a) 有位移電流 I 存在於電容器兩平行板間

(b) 在電容器中有時變電場 $\frac{\partial E}{\partial t} = \frac{I}{\epsilon_0 A}$ 存在

(c) 在電容器兩平行板間距離中心軸 r ($r < R$) 處有大小為

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

(d) 在電容器中的電流是因磁場隨時間變化感應產生的

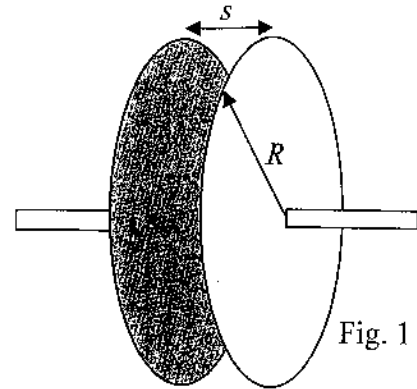


Fig. 1

2. 欲堆積一半徑為 R ，體電荷密度為 ρ 的實心球形電荷所需作的功為

(a) $\frac{8\pi\rho^2 b^5}{15\epsilon_0}$, (b) $\frac{4\pi\rho^2 b^5}{15\epsilon_0}$, (c) $\frac{3\pi\rho^2 b^5}{5\epsilon_0}$, (d) $\frac{3\pi\rho^2 b^5}{20\epsilon_0}$

3. A current I flows in the inner conductor of an infinitely long coaxial line and returns via the outer conductor. The radius of the inner conductor is a , and the inner and outer radii of outer conductor are b and c , respectively. (a) Find the magnetic flux density B for all regions. (b) Calculate the energy stored in the magnetic field of the region $a < r < b$. (16%)

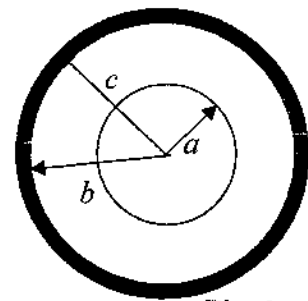


Fig. 2

4. Two charges ($+q$ and $+q$) are arranged along the z -axis at $z=d/2$ and $z=-d/2$, respectively. Determine the electric potential V and electric field E at a distance point $P(R, \theta, \phi)$. ($R \gg d$) (20%)

5. An infinitely long square metal pipe (sides π) is grounded, but one end, at $z=0$, is maintained at a specified potential V_0 , as indicated in Fig. 3. Find the potential inside the pipe. (20%)

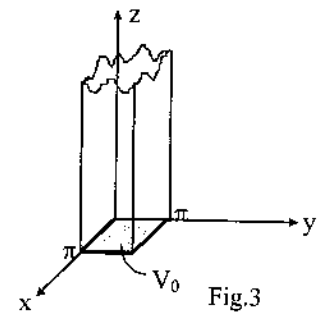


Fig. 3

6. Two dielectric media with permittivities ϵ_1 and ϵ_2 are separated by a charge free boundary. The electric field intensity in medium 1 at the point P_1 has a magnitude E_1 (as shown in Fig. 4) and makes an angle α_1 with the normal. Determine the magnitude and direction of the electric field intensity at point P_2 in medium 2. (14%)

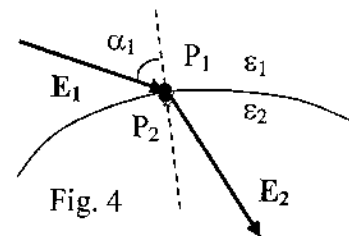


Fig. 4