10% for each problem.

1.(a) $\oint_c \frac{dz}{z^2+1}, c:|z+i|=1, unit-circle, counterclockwise$

(b) $\oint_c \frac{dz}{z^2+1}, c:|z|=2, circle, counterclockwise$

2. $\int_{-\infty}^{\infty} \frac{dx}{x^4+16}$

3. $\int_{-\infty}^{\infty} \frac{dx}{x^2-2ix}$

4.Find the Laurent series of $\frac{z^2-4}{(z-1)^2}$ with center $z_0=1$, converge at $0<|z-z_0|<R$,and determine the convergent region R.

5.Find the eigen value and eigen vectors of $\begin{bmatrix} 4 & -6 & -6 \\ 0 & -2 & 0 \\ 1 & -1 & -1 \end{bmatrix}$.

6.Solve the differential equation $y'' + 2y' + 10y = 25x^2 + 3$.

7. Solve the differential equation by the Laplace transform

$y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$.

8.Find divergence and curl of vector $\vec{v} = (x^2+y^2+z^2)^{-3/2}(x\hat{i} + y\hat{j} + z\hat{k})$, and

div(curl $\vec{v}$).

9.Find the moment of inertia of a solid sphere of radius R and mass M about diameter.

(Need calculation in details).

10.Find eigenfrequencies for two coupled-oscillators

$U=\frac{1}{2}kx_1^2 + \frac{1}{2}k_{12}(x_2-x_1)^2 + \frac{1}{2}kx_2^2$ and $T=\frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}M\dot{x}_2^2$

1. If a projectile is fired due east from a point on the earth's surface at a northern latitude θ, with a velocity of magnitude V_0 and at an angle of inclination to the horizontal of β, find the lateral deflection when the projectile strikes the earth. Earth's spin is ω .(20%)

2. The interaction between two atoms in a diatomic molecule, m_1 and m_2 , is approximately described by the potential

$$V(x) = -ax^{-6} + bx^{-12}, \quad a>0,\ b>0$$

Where x is the distance between the atoms. Find the force between the Two atoms. Describe the possible motion qualitatively first, and then find the equilibrium distance and the period of small oscillations about the equilibrium distance. (20%)

3. A bead of mass m slides freely on a smooth circular wire of radius b that rotates in a horizontal plane about a point on the circle with a constant angular velocity ω(Fig. 1.).

(a) Determine the Lagrangian function and Lagrange's equations of motion.

(b) Find the reaction of the wire.

(20%)

4. Determine the oscillations of a system with two degrees of freedom whose Lagrangian is

$$L = \frac{1}{2}(\dot{x}^2+\dot{y}^2) - \frac{1}{2}\omega_0^2(x^2+y^2) + \beta xy.$$

(20%)

5. A dumbbell—two equal masses connected by a massless rigid rod of length a—rotates at a fixed inclination θ with constant angular velocity ω about a pivot at the center of the rod(Fig. 2.). Find the angular momentum and torque of the system. (20%)

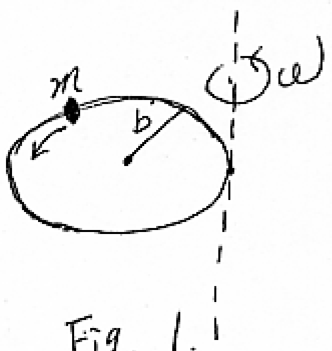


Fig. 1.

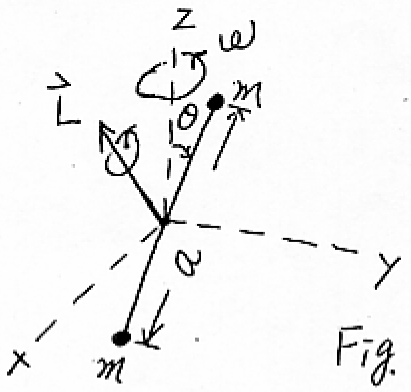
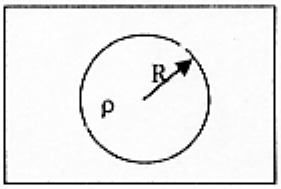
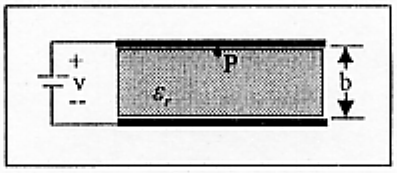


Fig. 2.

1. (25%)A sphere with a radius R contains charges distributed as $\rho = ar^2 + br + c$ where ρ and r are the charge density and the distance to the center of the sphere. Please calculate the electric field ($\vec{E}$) and the electric potential (V) at r < R, r = R and r < R.

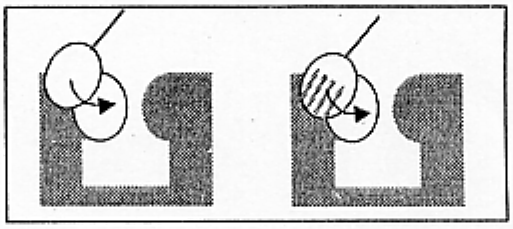


2. (25%)A linear dielectric material with a dielectric constant ε_r is placed inside a parallel capacitor which is connected to a DC battery with a voltage V (as shown in the right figure). When the capacitor is charged completely, what is the bounded surface charge on the surface P point of the dielectric material?



3. (25%)A copper ring of radius R is set spinning at angular velocity ω. Find the vector potential **A** and magnetic flux-density **B** at the center of the ring.

4. (25%)Please answer the following questions as detail as possible:
   (a) Two thin round Al plates are hung by a thin cord and swung through a horse-shoes magnet. Please answer what may happen to these two Al plates and why?



   (b) In static magnetism, $\vec{\nabla} \bullet \vec{B} = 0$ implies the non-existence of monopole. If someday it does be discovered to exist, the equation may rewrite to $\vec{\nabla} \bullet \vec{B} = \mu_0 \rho_B$. Please answer what will the "new" Maxwell equation be?