

(20%) 1. Find the solution of a vibrating two-dimensional rectangular membrane. The wave equation is

as follows:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

The boundary conditions are

$u(0,y,t)=0, u(a,y,t)=0, u(x,0,t)=0, u(x,a,t)=0$ , for all  $t$ , and initial conditions are

$$u(x,y,0)=f(x,y), \text{ and } \left. \frac{\partial u}{\partial t} \right|_{t=0} = g(x,y)$$

(20%) 2. Integrations a).  $\oint_c \frac{15z+9}{z^3-9z} dz$ ,  
 $c: |z+2+i|=3$ , counterclockwise

b).  $\int_0^{\infty} \frac{\cos x}{1+x^4} dx$

(20%) 3. Find eigenvalues and eigenvectors of the following matrices:

a).  $\begin{bmatrix} \cos \theta & e^{-i\theta} \sin \theta \\ e^{i\theta} \sin \theta & -\cos \theta \end{bmatrix}$

b).  $\begin{bmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{bmatrix}$

(20%) 4. Solve the following differential equations:

a).  $(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0$

b).  $3y'' + 10y' + 3y = 9x + 5 \cos x$

(20%) 5. Find the general solution of the system

$$y_1' = 6y_1 + 9y_2$$

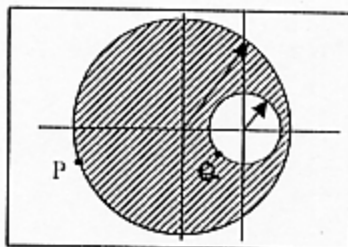
$$y_2' = y_1 + 6y_2$$

and determine the type and stability of the critical point.

1. 請用數學詳細說明三種 Damping Oscillation 的運動形式及其特性。並畫出三種運動隨時間的變化及各別的 Phase Diagram(10%)
2. a). 請詳細說明混沌系統的兩種特徵。(6%)  
b). 名詞解釋：(6%)  
Poincare Section, Lyapunov Exponent, Strange Attractor,  
c). 請說明下列兩種現象的物理原因：(6%)  
潮汐力(Tidal Force) 及 佛科擺(Foucault Pendulum)
3. 請用數學詳細說明：慣性座標中 "時間和空間的均勻性 (homogeneous) 及均向性(isotropic)" 和 "物理系統的守恆量" 之間的關係。(10%)
4. 請用數學詳細說明：在 Central Force Motion 中，我們如何把 "三度空間-兩個物體" 的問題，變成用 Effective Potential 討論的 "一度空間-單一粒子" 的問題。(10%)
5. 一個質量為  $m$ 、速度為  $u_1$  的粒子，正面撞擊另一個質量為  $2m$  的靜止粒子，若為非彈性碰撞而且使得總動能的損失為最多，求兩粒子碰撞後的速度。(7%)
6. 有一個三個粒子的偶合振盪系統，已知其動能為  

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}M\dot{x}_2^2 + \frac{1}{2}m\dot{x}_3^2$$
 位能為  $U = \frac{1}{2}k(x_2 - x_1)^2 + \frac{1}{2}k(x_3 - x_2)^2$ 。  
 a. 請寫出其 Lagrangian 並由 Lagrange's Equation 求出其運動方程式。(5%)  
 b. 請由 Lagrangian 求出其 Hamiltonian 並由 Hamilton's Equation 求出其運動方程式。(5%)  
 c. 請解出此系統的 Normal Frequencies 及 Normal Coordinates。(10%)  
 d. 請詳細說明此系統的 Normal Modes 的運動形式。(10%)
7. 有一個三個粒子的質點系統，各粒子的質量及位置座標如下：  
 $m_1 = 3M$  在  $(a, 0, a)$ ， $m_2 = 4M$  在  $(a, a, -a)$ ， $m_3 = 2M$  在  $(-a, a, 0)$ 。  
 請求出此系統的：a). Inertia Tensor, b). Principal Axes, c). Principal Moments of Inertia。(15%)

- 25/ 1. A sphere of a radius  $b$  with a constant charge density  $\rho$  contains a sphere cavity in it. The sphere cavity has a radius  $a$  and separates with the sphere ball a distance  $l$ . The  $a$ ,  $b$  and  $l$  are  $a < l < b$ . Please calculate  $\vec{E}$  and  $V$  on the surfaces of the large sphere, P point, and the cavity, Q point



- 10/ 2. (a) Show that the capacitance of a spherical capacitor of inner and outer radii  $R_1$  and  $R_2$  is  $C = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$  (b) If they both contain charge  $Q$  and  $-Q$ , how much the energy is stored in this capacitor?

- 15/ 3. A cylinder with radius  $a$  and length  $L$  carries a uniform magnetization  $M$  parallel to its axis. Find the vector potential  $A$ , magnetic flux-density  $B$  and the magnetic field  $H$  inside and outside of the cylinder.

- 20/ 4. A Monopolar Motor is assembled by conducting material made disk and cylindrical bar. Their diameters are shown in the right figure. An uniform magnetic field is supplied parallel to the cylindrical bar and perpendicular to the disk. When an external battery supplied a current  $I$  from the positive side into the bar and along the radius direction of the disk back to the negative side, please calculate the torque ( $\vec{\tau}$ ) and the angular velocity ( $\vec{\omega}$ ) of the disk. During the electric conduction, the electric current experiences an electric resistance of  $R$ .

