物訂三.

（橫書式） 國立中山大學八十八學年度轉學生招生考試試題

科目：應用數學（物理系三年級） 共1頁 第 頁

1. A force field is described by

$$\vec{F} = -\hat{i}\,\frac{y}{x^2+y^2} + \hat{j}\,\frac{x}{x^2+y^2}.$$

(a) Is $\vec{F}$ a conservative force? Why? (10)

(b) Calculate the work done by $\vec{F}$ in encircling the unit circle centered at the origin once counterclockwise. (10)

2. Given a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(a) Show that A is a Hermitian matrix. (5)

(b) Find the eigenvalues and the corresponding normalised eigenvectors. (10)

(c) Find a transformation matrix R, such that $R^{+}AR$ = a diagonal matrix. (5)

3. Given $y = x$ is a solution of

$$(x^2+1)y'' - 2xy' + 2y = 0, \quad y' = \frac{dy}{dx}$$

to find the general solution of $y(x)$. (20)

4. Expand $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$ in Fourier Series. (20)

5. Evaluate the Integration (20)

$$\int_0^\infty \frac{dx}{x^4+1} = ?$$

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科目：力 學（物理系三年級） 共一頁 第 頁

1. A simple pendulum of length l and mass m is released at θ_0. Assume that there is no resisting force and the vibration is small.

(a) Find the equation of motion $\theta(t)$.

(b) Find the period of this motion.

(c) Find $\theta(t)$ if the retarding force $2m\sqrt{gl}\,\dot{\theta}$ is taken into account. (20%)

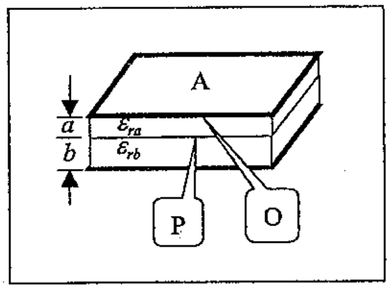
2. Obtain the Fourier expension of the function $F(t) = \begin{cases} +1 & -\frac{\pi}{\omega} < t < 0 \\ -1 & 0 < t < \frac{\pi}{\omega} \end{cases}$ (20%)

3. A mass m lies on the perpendicular through the center of a uniform thin circular plate of radius a and at distance b from the center. Find the force of attraction between the plate and the mass m. (20%)

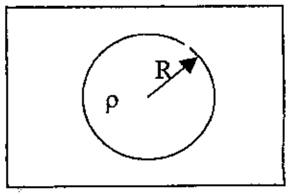
4. A particle of mass m starts at rest on top of a smooth fixed hemisphere of radius a. Find the force of constraint and determine the angle at which the particle leaves the hemisphere. (20%)

5. A particle of mass m is moving in a circle of radius r_0 under the action of an attractive force $F = -\frac{1}{r^2} e^{-\frac{r}{a}}$, where $a>0$. Find the condition that the circular motion has a stable orbit at radius r_0. (20%)

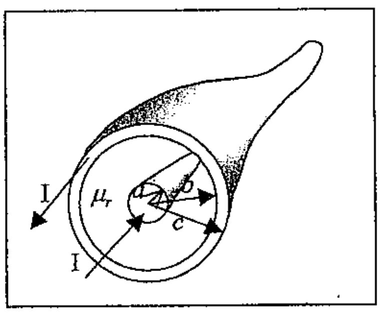
1. Two copper plates with surface area A are separated by two dielectric materials as shown in the right figure. The relative permittivities and the thickness of two dielectric materials are ε_{ra}, ε_{rb} and a, b respectively. Please find the electric displacement ($\vec{D}$), the electric field ($\vec{E}$), the polarization ($\vec{P}$), the capacitance (C), the surface charge density (σ_b) and the free charge density (σ_f) at O and P points.



2. A sphere with a radius R contains charges distributed as $\rho = ar^2 + br + c$ where ρ and r are the charge density and the distance to the center of the sphere. Please calculate the electric field ($\vec{E}$) and the electric potential (V) at r < R, r = R and r < R.



3. A coaxial cable carries a current I flowing in opposite directions in the inner core and in the outer cladding. Assuming the current is distributed uniformly over the conducting core and cladding, and a special linear magnetic material with a permeability of μ_r is sandwitched in between. Please calculate the flux density ($\vec{B}$) and the magnetic field ($\vec{H}$) in r > c, b < r < c, a < r < b and in r < a.



4. A Monopolar Motor is assembled by conducting material made disk and cylindrical bar. Their diameters are shown in the right figure. An uniform magnetic field is supplied parallel to the cylindircal bar and perpendicular to the disk. When an external battery supplied a current I from the positive side into the bar and along the radius direction of the disk back to the negative side, please calculate the torque ($\vec{\tau}$) and the angular velocity ($\vec{\omega}$) of the disk. During the electric conduction, the electric current experiences an electric resistance of R.