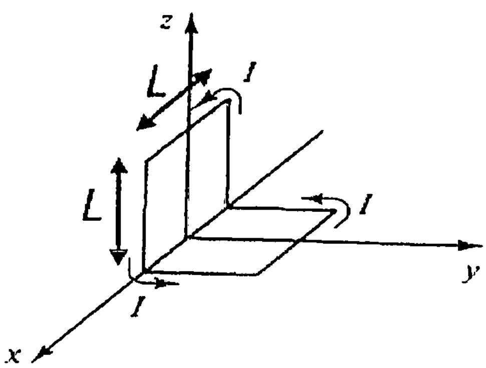
國立中山大學104學年度轉學考招生考試試題

科目名稱：電磁學【物理系三年級】　　　　　　題號：723003

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　共1頁第1頁

1. A point charge $+Q$ is situated a distance D from the center of a grounded conducting sphere of radius R. ($D > R$) Find the force between the charge and the sphere. (20 points)
2. Write down the Maxwell equations in vacuum. (no charge and no current in the space.) (20 points)
3. Proof that the speed of an electromagnetic wave in vacuum is light speed. (20 points)
4. Find the magnetic field a distance z above the center of a circular loop of radius R, which carriers a steady current I. (20 points)
5. As shown in the figure, calculate the magnetic dipole moment of the loop. (20 points)

(所有試題請寫出計算過程以及必須物理概念否則將酌予扣分.)



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每題皆為多選題、皆有五個選項，其中至少一個(至多五個)是正確答案，倒扣原則比照指考方式。第一到第二題，每題 8 分，第三到第八題，每題 14 分，共 100 分。

1. A and B are matrices. $(AB)^{-1} = ?$ (8 分)

   (a) $(AB)^T$
   (b) $A^{-1}B^{-1}$
   (c) $B^{-1}A^{-1}$
   (d) B^TA^T
   (e) A^TB^{-1}

2. A, B, and C are vectors. $(\mathrm{A} \times \mathrm{B}) \cdot \mathrm{C} = ?$ (8 分)

   (a) $\mathrm{A} \cdot (\mathrm{B} \times \mathrm{C})$
   (b) $\mathrm{C} \cdot (\mathrm{B} \times \mathrm{A})$
   (c) $\mathrm{B} \cdot (\mathrm{C} \times \mathrm{A})$
   (d) $(\mathrm{C} \times \mathrm{A}) \cdot \mathrm{B}$
   (e) $(\mathrm{C} \times \mathrm{B}) \cdot \mathrm{A}$

3. The differential equation describing the damped oscillation is

   $m\ddot{x} + b\dot{x} + kx = 0$

   which we can write as

   $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$

   Here $\beta \equiv b/2m$ and $\omega_0 = \sqrt{k/m}$. Find the general solution: (14 分)

   (a) $x(t) = e^{-\beta t}\left[A_1 e^{i\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{-i\sqrt{\beta^2-\omega_0^2}\,t}\right]$

   (b) $x(t) = e^{-\beta t}\left[A_1 e^{i\sqrt{\omega_0^2-\beta^2}\,t} + A_2 e^{-i\sqrt{\omega_0^2-\beta^2}\,t}\right]$

   (c) $x(t) = e^{\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}\,t}\right]$

   (d) $x(t) = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}\,t}\right]$

   (e) $x(t) = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{\sqrt{\beta^2+\omega_0^2}\,t}\right]$

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4. Consider the electrical circuit shown in Figure 1. (14 分)

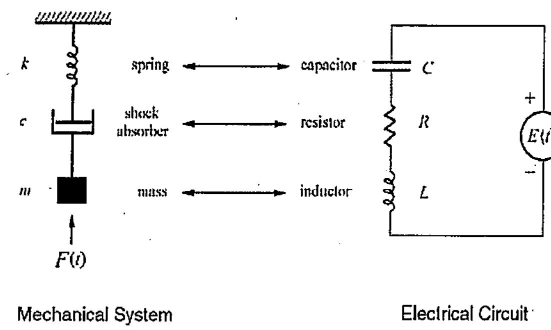

Figure 1

Write down the Kirchhoff's equation for the circuit: (given q as charge)

(a) $L\ddot{q} + C\dot{q} + Rq = E(t)$

(b) $\frac{1}{C}\ddot{q} + R\dot{q} + Lq = E(t)$

(c) $L\ddot{q} + R\dot{q} + \frac{1}{C}q = E(t)$

(d) $\frac{1}{L}\ddot{q} + \frac{1}{R}\dot{q} + \frac{1}{C}q = E(t)$

(e) $R\ddot{q} + L\dot{q} + Cq = E(t)$

5. Consider the shell in Figure 2. (14 分)

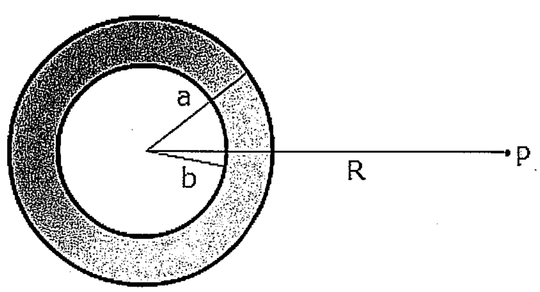

Figure 2

Calculate the gravitational potential everywhere: (given ρ as the density of the shell)

(a) $\Phi(R > a) = -\frac{4\pi\rho G}{3R}(a^3 - b^3)$

(b) $\Phi(R > a) = -\frac{2\pi\rho G}{R}(a^3 - b^3)$

(c) $\Phi(R < b) = -2\pi\rho G(a^2 - b^2)$

(d) $\Phi(R < b) = -\frac{4}{3}\pi\rho G(a^2 - b^2)$

(e) $\Phi(b < R < a) = -4\pi\rho G(\frac{a^2}{3} - \frac{b^3}{3R} - \frac{R^2}{6})$

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6. Consider a thin uniform disk of mass M, density ρ, and radius a (i.e., on x-y plane). Find the force and gravitational potential on a mass m located along the axis of the disk (i.e., z direction). (14 分)

(a) $\Phi(z) = -\frac{4}{3}\pi\rho G((a^2 + z^2)^{1/2} - z^2)$

(b) $\Phi(z) = -4\pi\rho G((a^2 + z^2)^{1/2} - z^2)$

(c) $F = 2\pi m\rho G\left[\frac{z}{(a^2+z^2)^{1/2}} - 1\right]$

(d) $F = 4\pi m\rho G\left[\frac{z}{(a^2+z^2)^{1/2}} - 1\right]$

(e) $F = \frac{4}{3}\pi m\rho G\left[\frac{z}{(a^2+z^2)^{1/2}} - 1\right]$

7. Based on Figure 3, using the Hamiltonian method, find the equations of motion for a spherical pendulum of mass m and length b. (14 分)

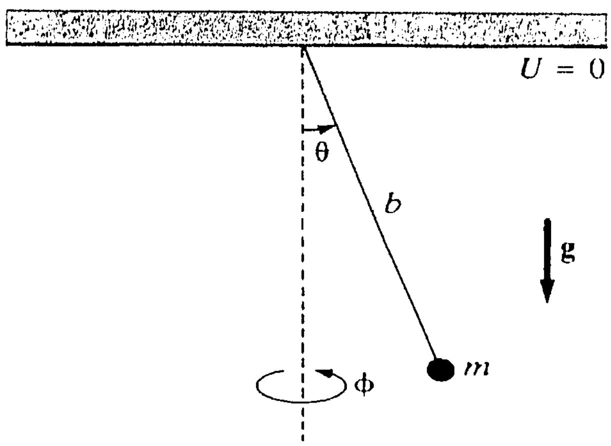

Figure 3

(a) $p_\theta = mb^2\dot{\theta}$

(b) $p_\phi = mb^2 \sin^2\theta\, \dot{\phi}$

(c) $\dot{p}_\phi = mb^2 \sin^2\theta\, \ddot{\phi}$

(d) $H = \frac{p_\theta^2}{2m} + \frac{p_\phi^2}{2m} + mgb\cos\theta$

(e) $H = \frac{p_\theta^2}{2mb^2} + \frac{p_\phi^2}{2mb^2\sin^2\theta} - mgb\cos\theta$

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科目名稱：力學【物理系三年級】　　題號：723004

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8. Assume a central-force field that allows a particle to move in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and α are constants. Find the force law and determine r(t) and θ(t). (Given the mass of the particle μ and $l = \mu r^2\dot{\theta}$)

   (Assume $\theta(0) = 0$) (14 分)

   (a) $F(r) = \frac{-l^2\alpha^2}{\mu r^3}$

   (b) $F(r) = \frac{-l^2}{\mu r^3}(\alpha^2 - 1)$

   (c) $\theta(t) = \frac{1}{\alpha}\ln\left(\frac{2\alpha l t}{\mu k}\right)$

   (d) $r(t) = \frac{2\alpha l t}{\mu}$

   (e) $r(t) = \left[\frac{2\alpha l t}{\mu} + k^2\right]^{1/2}$