

Do all problems in detail.

1. Let $f(x) = x \ln x + 2$.
- (a) Find $f'(1)$. [10%]
- (b) Find an equation of the tangent line of the graph of $y = f(x)$ at the point $(1, 2)$. [10%]

2. (a) Evaluate $\int_0^1 \frac{1}{e^{-x} + e^x} dx$. [10%]
- (b) Evaluate $\int_0^\infty (\sin t) e^{-st} dt, s > 0$. [10%]

3. Let $f(x, y) = 3 + 4x - x^2 - 2y - 2y^2 + 2xy$.
- (a) Find all relative maxima and minima of f . [10%]
- (b) Find the directional derivative of f at $(0, 1)$ in the direction $\langle 1, 2 \rangle$. [10%]

4. (a) Evaluate $\int_1^2 \int_1^4 \left(\frac{x}{y} + \frac{y}{x} \right) dy dx$. [10%]
- (b) Evaluate

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{\sqrt{x^2 + y^2}}{1 + x^2 + y^2} dx dy.$$

[10%]

5. (a) Evaluate the line integral $\int_C xy^3 dt$, where C is the curve $x = 4t, y = e^t, 0 \leq t \leq 1$. [10%]
- (b) Find the work done by the force field $\mathbf{F}(x, y) = xy \mathbf{i} - y^2 \mathbf{j}$ in moving a particle along the quarter-circle $\mathbf{r}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j}, 0 \leq t \leq \pi/2$. [10%]

1. A conical pendulum consists of a bob of mass m in motion in a circular path in a horizontal plane as shown in Figure 1. During the motion, the supporting wire of length ℓ maintains the constant angle θ with the vertical. Find the magnitude of the angular momentum of the bob about the center of the circle.

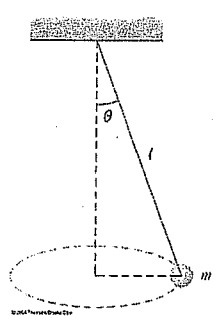


Figure 1

2. A frog in a hemispherical pod (Fig. 2) just floats without sinking into a sea of blue-green liquid with density 1.35 g/cm^3 . If the pod has radius 6.00 cm and negligible mass, what is the mass of the frog?

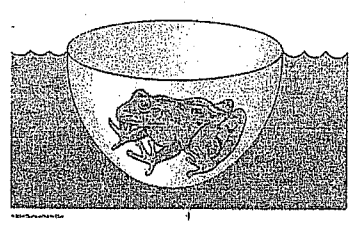


Figure 2

3. A large block P executes horizontal simple harmonic motion as it slides across a frictionless surface with a frequency $f = 1.50 \text{ Hz}$. Block B rests on it, as shown in Figure 3, and the coefficient of static friction between the two is $\mu_s = 0.600$. What maximum amplitude of oscillation can the system have if block B is not to slip?

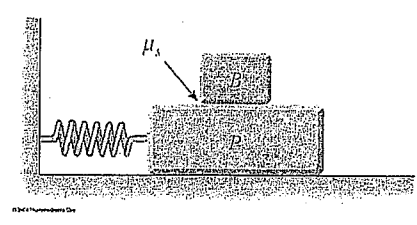


Fig. 3

4. Two waves that set up a standing wave in a long string are given by the wave functions

$$y_1 = A \sin(kx - \omega t + \phi)$$

and

$$y_2 = A \sin(kx + \omega t)$$

Show (a) that the addition of the arbitrary phase constant ϕ changes only the position of the nodes (5%), and in particular (b) that the distance between nodes is still one half the wavelength. (5%)

5. An ideal gas initially at P_i , V_i , and T_i is taken through a cycle as in Figure 5.

(a) Find the net work done on the gas per cycle. (5%)(b) What is the net energy added by heat to the system per cycle? (5%)

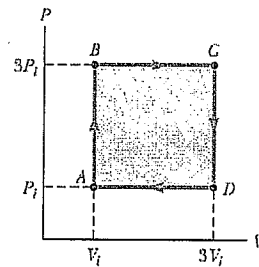


Figure 5

6. A sample consisting of n mol of an ideal gas undergoes a reversible isobaric expansion from volume V_i to volume $3V_i$. Find the change in entropy of the gas by calculating $\int_i^f dQ/T$ where $dQ = nC_p dT$.

7. Two identical particles, each having charge $+q$, are fixed in space and separated by a distance d . A third point charge $-Q$ is free to move and lies initially at rest on the perpendicular bisector of the two fixed charges a distance x from the midpoint between the two fixed charges (Fig. 7). Show that if x is small compared with d , the motion of $-Q$ will be simple harmonic along the perpendicular bisector. Determine the period of that motion.

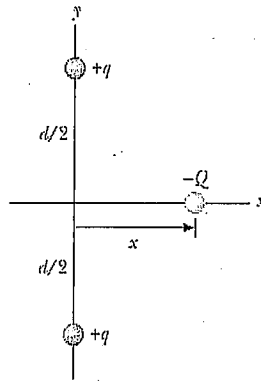


Figure 7

8. A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a tabletop as shown in Figure 8. (a) Determine the magnetic flux through the loop due to the current I . (5%) (b) Suppose the current is changing with time according to $I = a + bt$, where a and b are constants. Determine the emf that is induced in the loop if $b = 10.0 \text{ A/s}$. (5%)

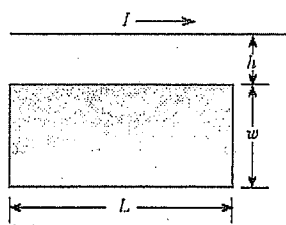


Figure 8

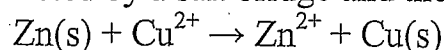
9. An oil film ($n = 1.45$) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find the wavelength of the light in the visible spectrum most strongly reflected.
10. Two wavelengths λ and $\lambda + \Delta\lambda$ (with $\Delta\lambda \ll \lambda$) are incident on a diffraction grating. Find the angular separation $\Delta\theta$ between the spectral lines in the m th-order spectrum. Assume d is the slit spacing and m is the order number.

請注意：

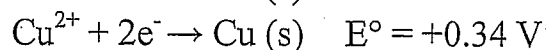
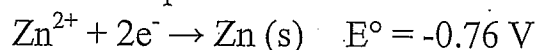
- (1) 考題中若涉及計算，請將演算過程列出，否則該題不予計分。
- (2) $\log 2 = 0.30$, $\log 3 = 0.48$
- (3) atomic weight: H=1, C=12, N=14, O=16, S=32
- (8%) 1. How many electrons can have the following quantum numbers in an atom: (a) $n = 2, l = 1$ (b) $n = 4, l = 2, m_l = -2$ (c) $n = 2$
(d) $n = 3, l = 2, m_l = +1$?
- (8%) 2. Predict the ground-state electron configuration of an atom of
(a) calcium; (b) nitrogen; (c) sodium; (d) iron.
- (8%) 3. In which of the following compounds do the bonds have greater covalent character: (a) NaBr or (b) MgBr₂? Explain your answer.
- (8%) 4. List the carbon-oxygen bonds in the following compounds in order of increasing length: (a) CH₃CH₂OH; (b) H₂CO; (c) CO. Explain your answer.
- (8%) 5. Name each of the following as an unbranched alkane in English:
(a) C₂H₆; (b) C₆H₁₄; (c) C₈H₁₈; (d) C₇H₁₆.
- (10%) 6. The density of 0.35 M (NH₄)₂SO₄ (aq) is 1.027 g/mL.
Determine (a) the molality; (b) the mole fraction of ammonium sulfate in the solution.
- (6%) 7. Express the units for rate constants when the concentrations are in moles per liter and time is in seconds for (a) zero-order reactions;
(b) first-order reactions; (c) second-order reactions.
- (10%) 8. Calculate the pH and percentage protonation of a 0.20 M aqueous solution of methylamine, CH₃NH₂. The K_b for CH₃NH₂ is 3.2×10^{-4} .
- (10%) 9. Calculate the ratio of the molarities of CO₃²⁻ and HCO₃⁻ ions required to achieve buffering at pH = 10.0. The pK_{a2} of H₂CO₃ is 10.3.

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- (12%) 10. An electrochemical cell is created by placing a zinc electrode in a 1.00-molar solution of ZnSO_4 and placing a copper electrode in a 1.00-molar solution of CuSO_4 . The two compartments were connected by a salt bridge and the following reaction occurred at 25°C .



The reduction potentials:



- What is the standard potential for the cell?
 - What is the value of G° for the cell?
 - What is the value of K_{eq} for the reaction?
 - At a certain point in the progress of the reaction, $[\text{Cu}^{2+}]$ drops to 0.10-molar and $[\text{Zn}^{2+}]$ increases to 1.90-molar. What is the cell potential at this point?
- (12%) 11. $\text{H}_2\text{O(l)} \rightleftharpoons \text{H}_2\text{O(g)}$
- At 298K, the value of the equilibrium constant, K , for the reaction above is 0.036.
- What is the sign of ΔS° for the reaction above at 298K?
 - What is the sign of ΔH° for the reaction above at 298K?
 - What is the sign of ΔG° for the reaction above at 298K?
 - At approximately what temperature will ΔG° for the reaction be equal to zero?