

# 國立中山大學 97 學年度轉學生招生考試試題

科目：微積分【物理系二年級、資工系二年級、海工系二年級】

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共十題，每題10分。答題時，每題都必須寫下題號與詳細步驟。  
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Find the dimensions of the rectangle of maximum area  $A$  that can be inscribed in the portion of the parabola  $y^2 = 4px$  intercepted by the line  $x = a$  where  $p$  and  $a$  are positive constants.

2. If  $F = 1/r^2$  and  $F$  is measured as  $4 \pm 0.05$ , estimate  $r$ .

3. Evaluate the following limit:

$$\lim_{n \rightarrow \infty} \frac{\pi}{6n} \left[ \sec^2 \left( \frac{\pi}{6n} \right) + \sec^2 \left( \frac{2\pi}{6n} \right) + \cdots + \sec^2 \left( \frac{(n-1)\pi}{6n} \right) + \frac{4}{3} \right].$$

4. A solid has a base in the form of an ellipse with major axis 10 and minor axis 8. Find its volume if every section perpendicular to the major axis is an isosceles triangle with altitude 6.

5. Find the following integral:

$$\int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} dx.$$

6. Find the following integral:

$$\int \frac{dx}{2 + \cos x}$$

7. Find the Maclaurin series of  $\tan^{-1} x$  and evaluate the following sum

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

8. Find the interval of convergence of  $\sum_{n=1}^{\infty} \frac{x^n}{\ln(n+1)}$ .

9. Find the volume bounded by the paraboloid  $x^2 + y^2 = 4z$ , the cylinder  $x^2 + y^2 = 8y$ , and the plane  $z = 0$ .

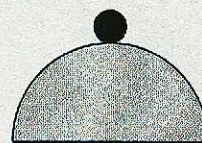
10. Find the integral of the function

$$f(x, y, z) = \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4}$$

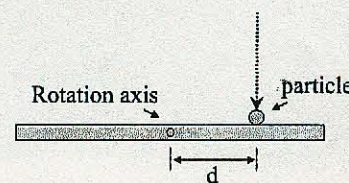
over the unit ball  $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$ .



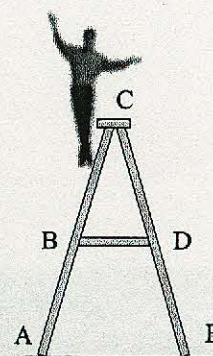
1. A small ball is initially seated on the top of a hemispherical ice mound of radius  $R = 15\text{ m}$ , shown on the right figure. It begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At what height does the small ball lose contact with the ice? (10%)



2. The figure on the right is an overhead view of a thin uniform rod of length  $0.600\text{ m}$  and mass  $M$  rotating horizontally at  $80.0\text{ rad/s}$  counterclockwise about an axis through its center. A particle of mass  $M/3.00$  and traveling horizontally at speed  $40.0\text{ m/s}$  hits the rod and sticks. The particle's path is perpendicular to the rod at the instant of the hit, at a distance  $d$  from the rod's center. At what value of  $d$  are rod and particle stationary after the hit? (10%)

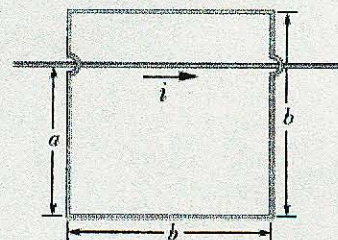


3. For the stepladder shown on the right, sides  $AC$  and  $CE$  are each  $2.44\text{ m}$  long and hinged at  $C$ . Bar  $BD$  is a tie-rod  $0.762\text{ m}$  long, halfway up. A man weighing  $854\text{ N}$  climbs  $1.80\text{ m}$  along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find (a) the tension in the tie-rod and the magnitudes of the forces on the ladder from the floor at (b)  $A$  and (c)  $E$ . (15%)



4. Explain in details how a Carnot engine works if it is placed between two heat reservoirs with temperatures of  $T_L$  and  $T_H$  ( $T_L < T_H$ ). Prove that its efficiency is  $1 - T_L/T_H$ . (15%)
5. A solid nonconducting sphere of radius  $R = 5.60\text{ cm}$  has a nonuniform charge distribution of volume charge density  $\rho = (14.1\text{ pC/m}^3)r/R$ , where  $r$  is radial distance from the sphere's center. (a) What is the sphere's total charge? What is the magnitude  $E$  of the electric field at (b)  $r = R/2.00$ , and (c)  $r = R$ ? (d) Sketch a graph of  $E$  versus  $r$ , for  $r = 0$  to  $r > 2.00R$ . (15%)

6. For the wire arrangement in the figure on the right,  $a = 12.0\text{ cm}$  and  $b = 16.0\text{ cm}$ . The current in the long straight wire is given by  $i = 4.50 t^2 - 10.0 t$ , where  $i$  is in amperes and  $t$  is in seconds. (a) Find the emf in the square loop at  $t = 3.00\text{ s}$ . (b) What is the direction of the induced current in the loop? (15%)



7. A long solenoid of radius  $R$  has  $n$  turns of wire per unit length and carries a time-varying current that varies sinusoidally as  $I = I_{max} \cos \omega t$ , where  $I_{max}$  is the maximum current and  $\omega$  is the angular frequency of the AC source. Determine the magnitudes of the induced electric fields (a) outside the solenoid, a distance  $r > R$  from its long central axis, (b) inside the solenoid, a distance  $r < R$  from its axis. (10%)

8. For a spherical concave mirror, where are the positions of the corresponding image if the object moves from very far away to very near the surface of the mirror? (10%)



# 國立中山大學 97 學年度轉學生招生考試試題

科目：普通化學【海資系二年級、海工系二年級】

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選擇題 (均為單選，每題 4 分，答錯倒扣 1 分，未作答則不計分亦不倒扣。)

1. ( ) Which of the following elements is diamagnetic ?  
(A) H (B) Li (C) Be (D) B (E) C
2. ( ) Which of the following rules states that no two electrons in an atom can have the same set of quantum numbers ?  
(A) Hund's rule (B) The Heisenberg Uncertainty principle (C) The Pauli Exclusion principle (D) The de Broglie hypothesis (E) The Bohr model
3. ( ) Which of the following is true of the alkali metal elements ?  
(A) They usually take the +2 oxidation state. (B) They have oxides that act as acid anhydrides.  
(C) They form covalent bonds with oxygen. (D) They are generally found in nature in compounds. (E) They have relatively large first ionization energies.
4. ( ) Which of the following ions has the smallest ionic radius ?  
(A)  $O^{2-}$  (B)  $F^{-}$  (C)  $Na^{+}$  (D)  $Mg^{2+}$  (E)  $Al^{3+}$
5. ( ) Which of the following is an impossible set of quantum numbers ( $n, l, m_l, m_s$ ) ?  
(A) 4, 0, 0,  $\frac{1}{2}$  (B) 4, 0, 1,  $\frac{1}{2}$  (C) 4, 1, 0,  $\frac{1}{2}$  (D) 4, 1, 1,  $\frac{1}{2}$   
(E) 4, 2, 1,  $\frac{1}{2}$
6. ( ) Which of the following species does NOT have a tetrahedral structure ?  
(A)  $CH_4$  (B)  $NH_4^{+}$  (C)  $SF_4$  (D)  $AlCl_4^{-}$  (E)  $CBr_4$
7. ( ) In which of the following species does the central atom NOT form  $sp^2$  hybrid orbitals ?  
(A)  $SO_2$  (B)  $BF_3$  (C)  $NO_3^{-}$  (D)  $SO_3$  (E)  $PCl_3$
8. ( ) Which of the molecules listed below has the largest dipole moment ?  
(A)  $Cl_2$  (B)  $HCl$  (C)  $SO_3$  (D)  $NO$  (E)  $N_2$
9. ( ) A gaseous mixture at  $25^{\circ}C$  contained 1 mole of  $CH_4$  and 2 moles of  $O_2$  and the pressure was measured at 2 atm. The gases then underwent the reaction shown below.  
 $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(g)$   
What was the pressure in the container after the reaction had gone to completion and the temperature was allowed to return to  $25^{\circ}C$  ?  
(A) 1 atm (B) 2 atm (C) 3 atm (D) 4 atm (E) 5 atm
10. ( ) A 22.0 gram sample of an unknown gas occupies 11.2 liters at standard temperature and pressure. Which of the following could be the identity of the gas ?  
(A)  $CO_2$  (B)  $SO_3$  (C)  $O_2$  (D)  $N_2$  (E) He
11. ( ) Which of the following conditions would be most likely to cause the ideal gas laws to fail ?  
I. High pressure  
II. High temperature  
III. Large volume  
(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

【背面還有試題】



# 國立中山大學 97 學年度轉學生招生考試試題

科目：普通化學【海資系二年級、海工系二年級】

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12. ( ) Which of the following gases would be expected to have a rate of effusion that is three times as large as that of  $H_2$  ?  
 (A)  $O_2$  (B)  $N_2$  (C) He (D)  $H_2O$  (E)  $CO_2$
13. ( ) Which of the following is true of a substance in equilibrium in the liquid phase ?  
 (A) Its temperature must be less than  $100^\circ C$ . (B) Its temperature must be greater than  $0^\circ C$ . (C) Its temperature must be lower than that of the surrounding atmosphere. (D) Its vapor pressure must be greater than the pressure of the surrounding atmosphere. (E) Its vapor pressure must be lower than the pressure of the surrounding atmosphere.
14. ( )  $2Al(s) + 3Cl_2(g) \rightarrow 2AlCl_3(s)$   
 The reaction above is not spontaneous under standard conditions but becomes spontaneous as the temperature decreases towards absolute zero. Which of the following is true at standard conditions ?  
 (A)  $\Delta S$  and  $\Delta H$  are both negative. (B)  $\Delta S$  and  $\Delta H$  are both positive.  
 (C)  $\Delta S$  is negative and  $\Delta H$  is positive. (D)  $\Delta S$  is positive and  $\Delta H$  is negative.  
 (E)  $\Delta S$  and  $\Delta H$  are both equal to zero.
15. ( ) If an endothermic reaction is spontaneous at 298K, which of the following must be true for the reaction ?  
 I.  $\Delta G$  is greater than zero.  
 II.  $\Delta H$  is greater than zero.  
 III.  $\Delta S$  is greater than zero.  
 (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III
16. ( )  $H_2(g) + F_2(g) \rightarrow 2HF(g)$   
 Gaseous hydrogen and fluorine combine in the reaction above to form hydrogen fluoride with an enthalpy change of  $-540 kJ$ . What is the value of the heat of formation of  $HF(g)$  ?  
 (A)  $-1080 kJ/mol$ . (B)  $-540 kJ/mol$ . (C)  $-270 kJ/mol$ . (D)  $270 kJ/mol$ . (E)  $540 kJ/mol$ .
17. ( ) Which of the following aqueous solutions has the highest boiling point ?  
 (A)  $0.5m NaCl$  (B)  $0.5m KBr$  (C)  $0.5m CaCl_2$  (D)  $0.5m C_6H_{12}O_6$  (E)  $0.5m NaNO_3$
18. ( ) A student added 1 liter of a  $1.0 M KCl$  solution to 1 liter of a  $1.0 M Pb(NO_3)_2$  solution. A lead chloride precipitate formed and nearly all of the lead ions disappeared from the solution. Which of the following lists the ions remaining in the solution in order of decreasing concentration ?  
 (A)  $[NO_3^-] > [K^+] > [Pb^{2+}]$  (B)  $[NO_3^-] > [Pb^{2+}] > [K^+]$  (C)  $[K^+] > [Pb^{2+}] > [NO_3^-]$   
 (D)  $[K^+] > [NO_3^-] > [Pb^{2+}]$  (E)  $[Pb^{2+}] > [NO_3^-] > [K^+]$
19. ( ) The solubility of  $PbS$  in water is  $3 \times 10^{-14}$  molar. What is the solubility product constant,  $K_{sp}$ , for  $PbS$  ?  
 (A)  $2 \times 10^{-7}$  (B)  $9 \times 10^{-7}$  (C)  $3 \times 10^{-14}$  (D)  $3 \times 10^{-28}$  (E)  $9 \times 10^{-28}$
20. ( ) For a particular salt, the solution process is endothermic. As the temperature at which the salt is dissolved increases, which of the following will occur ?  
 (A)  $K_{sp}$  will increase and the salt will become more soluble. (B)  $K_{sp}$  will decrease and the salt will become more soluble. (C)  $K_{sp}$  will increase and the salt will become less soluble. (D)  $K_{sp}$  will decrease and the salt will become less soluble. (E)  $K_{sp}$  will not change and the salt will become more soluble.



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科目：普通化學【海資系二年級、海工系二年級】

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21. ( ) Which of the following expressions is equal to the hydrogen ion concentration of a 1-molar solution of a very weak monoprotic acid, HA, with an ionization constant  $K_a$  ?  
(A)  $K_a$  (B)  $K_a^2$  (C)  $2K_a$  (D)  $2K_a^2$  (E)  $\sqrt{K_a}$

22. ( ) A multistep reaction takes place by the following mechanism ?



Which of the species shown above is an intermediate in the reaction ?

(A) A (B) B (C) C (D) D (E) E

23. ( )  $A+B \rightarrow C+D$

The rate law for the hypothetical reaction shown above is as follows:

$$\text{Rate} = k [A]$$

Which of the following changes to the system will increase the rate of the reaction ?

- I. An increase in the concentration of A.
- II. An increase in the concentration of B.
- III. An increase in the temperature.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

24. ( ) Which of the following is true of the oxidation-reaction that takes place in a galvanic cell under standard conditions ?

- (A)  $G^\circ$  and  $E^\circ$  are positive and  $K_{eq}$  is greater than 1.
- (B)  $G^\circ$  is negative,  $E^\circ$  is positive, and  $K_{eq}$  is greater than 1.
- (C)  $G^\circ$  is positive,  $E^\circ$  is negative, and  $K_{eq}$  is less than 1.
- (D)  $G^\circ$  and  $E^\circ$  are negative and  $K_{eq}$  is greater than 1.
- (E)  $G^\circ$  and  $E^\circ$  are negative and  $K_{eq}$  is less than 1.

25. ( ) A sample of radioactive material undergoing nuclear decay is found to contain only potassium and calcium. The sample could be undergoing which of the following decay processes ?

- I. Beta ( $\beta^-$ ) decay .
- II. Alpha decay .
- III. Electron capture .

(A) I only (B) II only (C) I and III only (D) II and III only  
(E) I, II, and III