Do all problems in detail.

(1)

(a) [10%] Evaluate $\int_0^\infty e^{-st}\sin at\,dt$ for $a, s > 0$.

(b) [10%] Evaluate $\int_0^2 x^3\sqrt{4-x^2}\,dx$.

(2)

(a) [15%] Determine whether the series $\sum_{n=1}^\infty \frac{n(n+1)}{(n+2)^3}$ is convergent or divergent.

(b) [10%] Find the radius of convergence of the power series $\sum_{n=1}^\infty \frac{x^n}{n^2+1}$.

(3)

(a) [10%] Find all local extrema of $f(x,y) = x^2 - xy + y^2 + 2x + 2y + 1$.

(b) [10%] Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{z^3}{\sqrt{x^2+y^2}}\,dz\,dy\,dx$.

(4)

(a) [10%] Is the vector field $\mathrm{f}(x,y) = (x^2 + xy^2, x^2y + y^2)$ conservative?

(b) [15%] A particle is moving along the parabola $y = x^2 + x - 1$ subject to the vector field given as in (a). Find the work done in moving from the point $(-1,-1)$ to the point $(3,11)$ if forces are measured in newtons and distances are measured in meters?

(c) [10%] Evaluate the surface integral $\int\int_S (x - 2x^3 + 3x^5)\,d\sigma$ where S is the part of the surface $z = x^2$ lying over $\{(x,y) : -1 \le x \le 1, 0 \le y \le 2\}$.

選擇題, please select **the best one** answer, each 2%

1. Which of the intermolecular forces is the most important contributor to the high surface tension shown by water? (1) dipole-dipole forces (2) dispersion forces (3) hydrogen bonding (4) ion-dipole forces
2. Which of the following forms an ionic solid? (1) Ag (2)$C_7H_4NH_2$ (3) RbI (4)SO_3
3. The highest coordination number for spherical packing is found in the (1) body-centered cubic structure.(2) simple cubic structure. (3) body-centered cubic and face-centered cubic. (4) cubic closest-packing and hexagonal closest packing
4. What is the chemical symbol for tin? (1) Fe (2) Sn (3) Ti(4) Zn
5. An astronaut uses a laboratory balance and weighs an object on earth and again on the moon. Which statement below about the weight and mass of the object is TRUE? (1) The mass and weight will be identical on the earth and the moon. (2) The mass will be the same on earth and moon but the weight will be less on the moon. (3) The weight will be the same on earth and moon but the mass will be less on the moon. (4) Both the mass and weight will be different on earth and moon.
6. The atoms of a particular element all have the same number of protons as neutrons. Which of the following must be true? (1) The atomic weight must be a whole number. (2) The mass number for each atom must equal the atomic weight of the element. (3) The mass number must be exactly twice the atomic number for each atom. (4) All of the above are true.
7. Which of the following has the least tendency to form an ion? (1) Ca (2) K (3) Kr (4) Se.
8. Which of the following statements about mass spectrometry is false? (1) Mass spectrometry can be used to determine the molecular weight of a compound. (2) The curvature of the path in a magnetic field is determined by the mass of the ion. (3) The paths of heavier ions are deflected more strongly than the paths of lighter ions. (4) The sample is changed into positively charged ions.
9. When silver nitrate reacts with barium chloride, silver chloride and barium nitrate are formed. How many grams of silver chloride are formed when 10 g of silver nitrate reacts with 15 g of barium chloride? (1) 8.44 g (2) 10.3 g (3) 20.6 g (4) 29.1 g
10. A student prepared a stock solution by dissolving 20.0 g of KOH in enough water to make 150 mL of solution. He took 15.0 mL of the diluted solution and added another 50.0 mL of deionized water. What is the concentration of the final solution? (1) 0.548 M (2) 0.713 M (3) 1.03 M (4) 1.66 M.
11. Which one of the following compounds is soluble in water? (1) Ag_3PO_4 (2) CuS (3) $Pb(NO_3)_2$ (4) $ZnCO_3$
12. Which of the following compounds is an Arrhenius base? (1) CH_3OH (2) CH_3COOH (3) HOCl (4) KOH
13. What reagent would distinguish between Ag^+ and Fe^{3+}? (1) $NaClO_3$ (2) NaI (3) $NaNO_3$ (4) NaOH.
14. For an orbital, a node is (1) the midpoint of the orbital (2) a surface inside which there is a 90% chance of finding the electron (3) a surface where there is a maximum probability of finding the electron (4) a surface where there is no chance of finding the electron.
15. How many electrons can a single orbital hold? (1) $2n$ (2) 2 (3) $2l+1$ (4) 8
16. Which element has the highest first ionization energy? (1) Ca (2) K (3) Li (4) Mg
17. Which of these elements has the highest first electron affinity? (1) Ca (2) N (3) Ne (4) S
18. The strongest reducing agent in the group 3A is (1) B (2) Al (3) Ga (4) Tl.
19. Which element has the lowest electronegativity? (1) Mg (2) Cl (3) Ca (4) Ba
20. How many lone pairs of electrons are on the P atom in PF_3 ? (1) 0 (2) 1 (3) 2 (4) 3.

21. What is the shape of BrF_4^-? (1) see-saw (2) square planar (3)square pyramidal (4) tetrahedral
22. What hybridization would be expected for the central atom in a molecule with square planar geometry? (1) sp^2 (2)sp^3 (3)sp^3d (4)sp^3d^2
23. Which of the following can be interpreted as a measure of randomness? (1) enthalpy (2)entropy (3) free energy (4) temperature.
24. The first law of thermodynamics (1) defines chemical energy. (2) defines entropy. (3) is a statement of conservation of energy. (4) provides a criterion for the spontaneity of a reaction.
25. Carbon dioxide is a gas which causes environmental concern because of the greenhouse effect. What is the approximate percentage (by volume) of CO_2 in the atmosphere? (1) less than 0.1% (2)about 1% (3)about 5% (4) 21%
26. An unknown gas effuses 2.3 times faster than N_2O_4 at the same temperature. What is the identity of the unknown gas? (1) CN_2 (2) NH_3 (3) N_2O (4) O_3
27. Which has the smallest dipole-dipole forces? (1) CH_3Cl (2) HCl (3) N_2 (4) CO
28. The change in the Gibbs free energy for dissolving solute in a saturated solution is (1) negative. (2) zero. (3) positive. (4) positive at low temperatures and negative at high temperatures.
29. Which concentration varies with temperature? (1) molality (2) molarity (3)mole fraction (4) wt%
30. What is the overall reaction order for the reaction represented by the rate law: Rate = $k[H_2][NO]^2$?(1)zero (2)first (3)second (4)third.
31. Which statement about the equilbrium constant is true? The value of Kc (1) changes as product concentration changes. (2) changes as reactant concentration changes.(3)changes as temperature changes.(4)never changes.
32. Which of the following changes in reaction conditions will not alter the equilibrium concentrations? (1) Addition of an inert gas to the reaction mixture. (2) Addition of reactants or products. (3) Decreasing the pressure or volume. (4) Increasing the temperature.
33. Commercially oxygen is usually obtained by (1) decomposition of mercury (II) oxide. (2) decomposition of potassium chlorate. (3) electrolytic decomposition of water. (4) fractional distillation of air.
34. Which of the following chemical reagent is not used in the purification of water (1) $NaNO_3$ (2) Cl_2 (3) CaO (4) $Al_2(SO_4)_3$
35. What is the strongest acid among the following? (1) CH_3COOH (2) $ClCH_2COOH$ (3) $Cl_2CHCOOH$ (4) Cl_3CCOOH
36. Which one of the following is not considered to be a Lewis base (1) H_2O (2) NH_3 (3) NH_4^+ (4) Cl^-
37. Which of the following sets of compounds is not characteristic of a buffer? (1) NaH_2PO_4 with Na_2HPO_4 (2) NH_3 with NH_4Cl (3) CH_3COOH with CH_3COONa (4) HNO_3 and $NaNO_3$
38. What is the most soluble salt of the following set? (1) $Ba(OH)_2$ with $K_{sp} = 5.0 \times 10^{-3}$ (2) $Ca(OH)_2$ with $K_{sp} = 4.7 \times 10^{-6}$ (3) $Cd(OH)_2$ with $K_{sp} = 5.3 \times 10^{-15}$ (4) $Fe(OH)_2$ with $K_{sp} = 2.5 \times 10^{-37}$
39. The Boltzmann formula is $S = k \ln W$. A perfect crystal has a molar entropy of 0 at absolute zero because (1) $W = 0$. (2) $W = 1$. (3) $W = N_A$ (4) $k = 1$.
40. Which provides the greatest increase in entropy? (1) H_2O (s, 0°C) → H_2O (l, 0°C) (2) H_2O (l, 0°C) → H_2O (l, 25°C) (3) H_2O (g, 0.1°C) → H_2O (s, 0.1°C) (4) H_2O (l, 100°C) → H_2O (g, 100°C)
41. If the cell reaction involves ions in solution, as the cell reaction in a galvanic cell continues, (1) E for the cell increases. (2) E for the cell decreases. (3) E° for the cell increases. (4) E° for the cell decreases.
42. When a cell reaction reaches equilibrium,(1) $E^\circ = 0$. (2) $E = 0$. (3) both E° and $E = 0$. (4) neither E° or $E = 0$.
43. Which carbon containing compound is an inorganic compound? (1) $C_7H_7NO_2$ (2) CO_2 (3) CH_3COOH (4) C_6H_5CN
44. Which of the following is the strongest acid? (1) HClO (2) $HClO_2$ (3) $HClO_3$ (4) $HClO_4$
45. What is the strongest oxidizing agent of the following set? (1) $MnCl_2$ (2) $Mn(OH)_3$ (3) MnO_2 (4) $KMnO_4$
46. Which substance is a good conductor? (1) carbon (2) germanium (3) gold (4) phosphorus
47. Which substance is a good semiconductor? (1) diamond (2) germanium (3) gold (4) phosphorus
48. How many isomers are there of C_5H_{12}? (1) 3 (2) 4 (3) 5 (4) 6
49. Which one of the following compounds is not a possible product of the reaction of methane with chlorine gas? (1) CH_3Cl (2) CH_2Cl_3 (3) $CHCl_3$ (4) CCl_4
50. Which amino acid has the simplest structure? (1) alanine (2) glycine (3) leucine (4) valine