國立中山大學八十九學年度 轉學生招生考試試題

科　目：微積分(物理系二年級、電機系二年級、資工系二年級、海工系二年級)　共　頁 第　頁

(一) 填充題：每個空格4分，共計60分. 此部份祇需將答案寫在答案卷上，並在答案前標明每個空格的英文字母代號；不需列出計算過程

1. Evaluate the following limits :

(i) $\lim_{x\to 0} \dfrac{2\cos x - 2 + x^2}{x^2 - \ln(1+x^2)} =$ (A)　　(ii) $\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{1}{\sqrt{4n^2-k^2}} =$ (B)

2. Evaluate the following integrals :

(i) $\int_1^2 x\sqrt{2-x}\,dx =$ (C)　　(ii) $\int_0^1 \{\int_{\sqrt{x}}^1 \dfrac{dy}{1+y^3}\}\,dx =$ (D)

3. Let p and q be two real numbers, and let $f(x) = e^{x^2-1} + px + q$ for all $x \in \mathbb{R}$. If $f(1) = 4$ is a local extremum of f, then $p =$ (E), $q =$ (F)

4. The length of the curve $y = \ln\cos x \quad (0 \le x \le \frac{\pi}{4})$, is (G)

5. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a function. Assume that f is differentiable at the point $(1,2) \in \mathbb{R}^2$ with $f(1,2) = 0$, $f_x(1,2) = -1$ and $f_y(1,2) = 3$.

(i) The equation of the tangent plane to the surface $z = f(x,y)$ at the point $(1,2,0)$ is (H)

(ii) If $\vec{u} = (\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$, then the directional derivative of f at $(1,2)$ in the direction $\vec{u}$ is (I)

(iii) If $g(t) = f(1 - 4\tan^{-1} t,\ 3 - e^{2t})$ for $t \in \mathbb{R}$, then $g'(0) =$ (J)

6. If $\Omega = \{(x,y) \in \mathbb{R}^2 : 1 \le x \le 2,\ 0 \le y \le 1\}$, then

$$\iint_\Omega xe^{xy}\,dx\,dy = \text{(K)}$$

7. If

$$f(x) = 3 + \int_2^{\sqrt{x}} \frac{dt}{1+3t^2+t^4} \quad \text{for } x > 0,$$

and if g is the inverse of f, then $f'(x) =$ (L), and $g'(3) =$ (M)

8. The interval of convergence of the power series $\sum_{k=3}^{\infty} \frac{\ln k}{k}(x-2)^k$ is (N)

9. If $f(x) = \cos(x^2)$, then $f^{(12)}(0) =$ (O)

(二) 計算題：請詳列計算過程，否則不計分

I. Let

$$f(x) = \begin{cases} \frac{1-\cos x}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Prove that f is differentiable on $\mathbb{R}$, and find $f'(x)$. (10 %)

II. Evaluate the value of the integral :

$$\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)} \quad (10\ \%)$$

III. Let $\Omega = \{(x,y) : x^2 + y^2 \le \frac{1}{4} \text{ and } x \ge 0\}$. Evaluate the double integral :

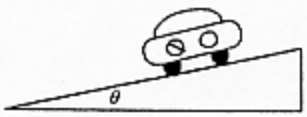
$$\iint_\Omega \sin^{-1}(x^2+y^2)\,dx\,dy \qquad (10\ \%)$$

IV. (i) If $g(t) = t^2 - \frac{t^4}{3} - \sin^2 t$, prove that $g(t) \le 0$ for all $t \in \mathbb{R}$. (5 %)

(ii) Use (i) to evaluate the limit : $\lim_{(x,y)\to(0,0)} \dfrac{\sin^2 x + \sin^2 y}{x^2+y^2}$ (5 %)

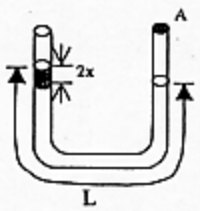
1. A balloon is rising at 10 m/s when its passenger throws up a ball at 12 m/s. How much later does the passenger catch the ball? (10%)

2. A car travels in a horizontal circle around a curve of radius r banked at an angle θ to the horizontal. If μ is the coefficient of static friction, find the maximum speed without sliding. (10%)



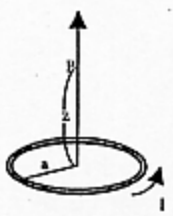
3. Find the center of mass of the flat semicircular plate of radius R. (15%)

4. The water is slightly displayed and then allowed to move freely. (15%)
   (a) Show that the liquid executes simple harmonic motion.
   (b) What is the period?



5. A charged disk of radius R with an uniform charge density σ over its surface. (20%)
   (a) What is the potential at a point P on the disk axis, a distance z from the disk?
   (b) What is the electric field at a point P derived from the electric potential directly?

6. Find the magnetic field at an arbitrary point P on the axis of a circular loop of radius a carrying current I. (15%)



7. An oscillating *LC* circuit consisting of a 1.0 *n*F capacitor and a 3.0 *mH* coil has a maximum voltage of 3.0 *V*.(15%)
   (a) What is the maximum charge on the capacitor?
   (b) What is the maximum current through the circuit?
   (c) What is the maximum energy stored in the magnetic field of the coil?

共 25 題選擇題（單選），每題 4 分，不做答以 0 分計，答錯倒扣 1 分。

1. 1 μm (micrometer) is ①1000 m ②1 m ③$10^{-3}$ m ④$10^{-6}$ m ⑤$10^{-9}$ m

2. The name of H^- is ①hydrogen ②hydride ③proton ④hydrogen minus ⑤hydrogen negative

3. A silicon chip used in an integrated circuit of a microcomputer has a mass of 5.68 mg. How many silicon (Si) atoms are present in the chip? (The atomic weight of Si is 28.08). ①0.202 ②$2.02\times10^{-4}$ ③$1.22\times10^{20}$ ④$1.22\times10^{23}$ ⑤$1.22\times10^{26}$

4. What volume of 16 M sulfuric acid must be used to prepare 1.5 L of a 0.10 N H_2SO_4 solution? ①9.4 L ②9.4 mL ③4.7 L ④4.7 mL ⑤none of the above

5. Which of the following reactions is not a redox reaction?
   ①$Ag^+ + Cl^- \rightarrow AgCl$
   ②$2Na + Cl_2 \rightarrow 2NaCl$
   ③$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
   ④$2S_2O_3^{2-} + I_3 \rightarrow S_4O_6^{2-} + 3I^-$
   ⑤none of the above

6. Calculate the equilibrium constant K at 25℃ for the reaction
   $3Tl^+_{(aq)} \rightarrow Tl^{3+}_{(aq)} + 2Tl_{(s)}$ $E^0 = -1.60V$

   ①$K=10^{-1.60}$ ②$K=10^{1.60}$ ③$K=10^{-27}$ ④$K=10^{27}$ ⑤none of the above

7. A sample of methane gas that has a volume of 5 L at 20 °C is heated to 40°C at constant pressure. Calculate its new volume. ①5 L ②5.34 L ③10 L ④10.34 L ⑤10.68 L

8. When a reaction results in the evolution of heat, it is said to be ①exothermic ②endothermic ③isothermic ④hyperthermic ⑤hypothermic

9. Which of the following ion (atom) has the largest radius? ①O^{2-} ②F^- ③Ne ④Na^+ ⑤Mg^{2+}

10. Which of the following electron transfer process between the different energy levels of hydrogen atom requires least energy? ①level n=1 to level n=2 ②n=2 to n=3 ③ n=3 to n=4 ④ n=4 to n=5 ⑤ n=5 to n=6 (level n=1: ground state, n=2: first excited state, etc.)

11. Which of the following is a possible set of quantum numbers for an electron?
①n= 2, ℓ= 1, m_ℓ= 0, m_s= 0
②n= 1, ℓ= 1, m_ℓ= 1, m_s= +1/2
③n= 3, ℓ= 2, m_ℓ= -3, m_s= +1/2
④n= 4, ℓ= 0, m_ℓ= 0, m_s= -1/2
⑤n= 1, ℓ= 1, m_ℓ= 1, m_s= +1/2

12. Which of the following molecule has the largest dipole moment? ①F_2 ②Br_2 ③HF ④HBr ⑤FBr

13. A buffer solution was prepared by mixing 1 L of 0.01 M HCOOH solution and 1L of 0.1 M HCOONa solution. The pH value of 4.7 was observed for this buffer solution. The pKa of HCOOH should be around ①5.7 ②4.7 ③3.7 ④0.47 ⑤47

14. What is the pH of a 0.1 N CH_3COOH solution? (pKa of CH_3COOH is 5) ①1 ②2 ③3 ④4 ⑤5

15. pKa value is a common way of describing the strength of an acid. Which one of the following acids has the largest pKa value? ① CH_3COOH ② CH_2FCOOH ③ $CH_2BrCOOH$ ④ CH_2FCOOH ⑤ $CHBr_2COOH$

16. Give the correct relationship between the change in standard free energy (ΔG^o) and the equilibrium constant (K) for a given reaction. ①ΔG^o=0, K=0 ②ΔG^o=0, K=1 ③ΔG^o=1, K=1 ④ΔG^o<0, K<0 ⑤ΔG^o>0, K>0

17. Which one is not an organic compound? ①CH_4 ②HCHO ③HCOOH ④H_2CO_3 ⑤CH_3NH_2

18. The name of polymer $(CH_2-CHCl)_n$ is ①PE ②PP ③PVC ④Teflon ⑤Polystyrene

19.For the reaction

$$A \rightarrow \text{Products}$$

we have found that the rate law is

$$\text{Rate} = -\Delta[A] / \Delta t = k[A]$$

If the initial concentration of A is 0.1M and 99 % of A has been consumed 20 minutes later, what is the half-life of this reactant A? ①1.0 min ②1.5 min ③2.0 min ④2.5 min ⑤3.0 min

20.For a general reaction involving a single reactant,

$$A \rightarrow \text{Products}$$

which is second order in A, the rate law is

$$\text{Rate} = -\Delta[A] / \Delta t = k[A]^2$$

The half-life of this reactant is ①$1/k$ ②$1/ k[A]_0$ ③$1/2k[A]_0$ ④$1/k[A]^2_0$ ⑤$1/2k[A]^2_0$

($[A]_0$ is the initial concentration of A)

21.A balloon is being inflated to is full extent by heating the air inside it. In the final stages of this process, the volume of the balloon changes from 4.00×10^6 L to 4.50×10^6 L by addition of 1.3×10^6 J of energy as heat. Assuming the balloon expends against a constant pressure of 1.0 atm, then, ΔE for this process is ①$1.3\times10^6$ J ②-1.8×10^8 J ③-8×10^7 J ④$1.8\times10^8$ J ⑤$8\times10^7$ J

(To convert between L×atm and J, use 1 L×atm=101.3 J)

22.At constant pressure, the change in enthalpy (ΔH) of a system is equal to ①q ②w ③q + w ④ΔE ⑤ΔE- q (ΔE represents the change in internal energy, q represents heat and w represents work.)

23.Which molecule has the lowest oxidation state in carbon?
①H_2CO_3 ②HCOOH ③HCHO ④CH_3OH ⑤HCOONa

24.Which of the following processes is called saponification?
①$RCOOR' + OH^- \rightarrow$
②$RCOOR' + H^+ \rightarrow$
③$RCOOH + OH^- \rightarrow$
④$RCOOH + H^+ \rightarrow$
⑤$RCOOH + H_2O \rightarrow$

25.The reaction

$$A + B \underset{k_r}{\overset{k_f}{\rightleftharpoons}} C + D$$

has the forward rate constant k_f and reverse rate constant k_r. The equilibrium constant K of this reaction is ① k_fk_r ②$1/ k_fk_r$ ③ k_f/ k_r ④ k_r / k_f ⑤ k_f-k_r