

注意事項：10 題單選題，每題答對得 10 分，答錯倒扣 3 分，未作答得 0 分。

- The rate of depreciation $\frac{dV}{dt}$ of a machine is inversely proportional to the square of $t + 2$, where V is the value of the machine t years after it was purchased. The initial value of the machine was \$500,000, and its value decreased \$100,000 in the first year. Estimate its value after 6 years. Round your answer to the nearest integer.
(A) \$500,000 (B) \$275,000 (C) \$550,000 (D) \$475,000
(E) \$350,000
- All edges of a cube are expanding at a rate of 4 centimeters per second. How fast is the volume changing when each edge is 5 centimeters?
(A) $100 \text{ cm}^3/\text{sec}$ (B) $300 \text{ cm}^3/\text{sec}$ (C) $200 \text{ cm}^3/\text{sec}$
(D) $240 \text{ cm}^3/\text{sec}$ (E) $80 \text{ cm}^3/\text{sec}$
- Find the cubic function of the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and the coefficients a, b, c, d are real numbers, which satisfies the conditions given below.

Relative maximum: (3, 5)

Relative minimum: (5, 3)

Inflection point: (4, 4)

- (A) $\frac{1}{2}x^3 - 6x^2 + \frac{41}{2}x + 21$ (B) $\frac{1}{2}x^3 - 6x^2 + \frac{41}{2}x - 21$
(C) $-\frac{1}{2}x^3 + 6x^2 - \frac{45}{2}x + 22$ (D) $\frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x + 22$
(E) $\frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 22$
- Find the derivative of the function $y = \cosh^{-1} 7x$.
(A) $y' = \frac{7}{\sqrt{49x^2 + 1}}$ (B) $y' = \frac{7}{\sqrt{1 - 49x^2}}$ (C) $y' = -\frac{7}{49x^2 + 1}$
(D) $y' = \frac{7}{\sqrt{49x^2 - 1}}$ (E) $y' = \frac{7}{1 - 49x^2}$
 - Find the volume of the solid generated by rotating the circle $x^2 + (y - 9)^2 = 36$ about the x -axis.
(A) $324\pi^2$ (B) $108\pi^2$ (C) 648π (D) 108π (E) $648\pi^2$
 - Evaluate $\int_0^{\frac{\pi}{4}} 4 \tan^3 x \, dx$.
(A) $2 - \ln 2$ (B) $2 + \ln 2$ (C) $2(1 - \ln 2)$ (D) $2(1 + \ln 2)$
(E) $4(1 + \ln 2)$

7. Find the Maclaurian series for the function $f(x) = \frac{1}{\sqrt{25+x^2}}$.

(A) $\frac{1}{5} + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1) x^{2n}}{2^n (2n-1)! 5^{2n+1}}$

(B) $\frac{1}{5} + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1) x^{2n}}{2^n n! 5^{2n+1}}$

(C) $\frac{1}{5} + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1) x^{2n}}{2^n n! 5^{2n+1}}$

(D) $\frac{1}{5} + \sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) x^n}{n!}$

(E) $\frac{1}{5} + \sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) x^n}{5^n n!}$

8. Find the point on the curve $y = 3x^3 + 10x^2 + 4x$ at which the curvature K is zero.

(A) $x = -\frac{10}{3}$ (B) $x = \frac{9}{10}$ (C) $x = -\frac{9}{10}$ (D) $x = -\frac{10}{9}$

(E) $x = \frac{10}{9}$

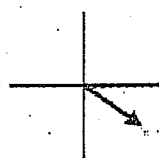
9. Find the highest point on the curve of intersection of the following surfaces: Cone: $x^2 + y^2 - z^2 = 0$, Plane: $x + 14z = 10$.

(A) $\frac{10}{13}$ (B) $\frac{13}{10}$ (C) 130 (D) 140 (E) 150

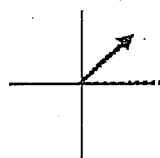
10. Find the volume of the solid between the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 = 36$, and inside the cone $z^2 = x^2 + y^2$.

(A) $\frac{182\pi}{3}(2 - \sqrt{2})$ (B) $182\pi(2 - \sqrt{2})$ (C) $\frac{182\sqrt{2}\pi}{3}$

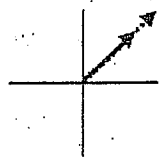
(D) $307\sqrt{2}\pi$ (E) $\frac{307\pi}{3}(2 - \sqrt{2})$

6. A 400-turn circular coil (radius = 1.0 cm) is oriented with its plane perpendicular to a uniform magnetic field which has a magnitude that varies sinusoidally with a frequency of 90 Hz. If the maximum value of the induced emf in the coil is observed to be 4.2 V, what is the maximum value of the magnitude of the varying magnetic field?
- a. 59 mT b. 62 mT c. 65 mT d. 68 mT e. 31 mT
7. A long solenoid has a radius of 2.0 cm and has 700 turns/m. If the current in the solenoid is decreasing at the rate of 8.0 A/s, what is the magnitude of the induced electric field at a point 2.5 cm from the axis of the solenoid?
- a. $56 \mu\text{V/m}$ b. $8.8 \mu\text{V/m}$ c. $88 \mu\text{V/m}$ d. $69 \mu\text{V/m}$ e. $44 \mu\text{V/m}$
8. A series LC circuit contains a 100 mH inductor, a 36.0 mF capacitor and a 12 V battery. The angular frequency of the electromagnetic oscillations in the circuit is
- a. $36.0 \times 10^{-4} \text{ rad/s}$.
 b. $6.00 \times 10^{-2} \text{ rad/s}$.
 c. 2.78 rad/s.
 d. 16.7 rad/s.
 e. 277 rad/s.
9. An inductor produces a back emf in a DC series RL circuit when a switch connecting the battery to the circuit is closed. We can explain this by
- a. Lenz's law.
 b. increasing magnetic flux within the coils of the inductor.
 c. increasing current in the coils of the inductor.
 d. all of the above.
 e. only (a) and (c) above.
10. The graphs below show the phasors ΔV_{max} and I_{max} for five RLC series circuits. The graph which represents a circuit where the capacitive reactance is greater than the inductive reactance is
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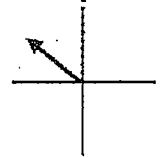
a.



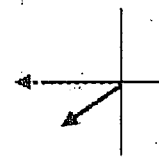
b.



c.



d.



e.
11. Monochromatic light ($\lambda = 500 \text{ nm}$) is incident on a soap bubble ($n = 1.40$). How thick is the bubble (in nm) if destructive interference occurs in the reflected light?
- a. 102 b. 179 c. 54 d. 1 e. 89
12. At what distance could one theoretically distinguish two automobile headlights separated by 1.5 meters? Assume a pupil diameter of 0.50 cm and yellow headlights seen at the wavelength $5.0 \times 10^{-7} \text{ m}$. Assume eye fluid has an average $n = 1.33$.
- a. 6.0 km b. 12 km c. 9.0 km d. 3.0 km e. 16 km

計算題 (40 分, 每題 10 分)

13. A Geiger counter is like an electroscope that discharges whenever ions formed by a radioactive particle produce a conducting path. A typical Geiger counter consists of a thin conducting wire of radius 0.002 cm stretched along the axis of a conducting cylinder of radius 2.0 cm. The wire and the cylinder carry equal and opposite charges of $8 \times 10^{-10} \text{ C}$ all along their length of 10.0 cm. What is the magnitude of the electric field at the surface of the wire?
14. Is it feasible to construct an air-filled parallel-plate capacitor that has its two plates separated by 0.10 mm and has a capacitance of 1.0 F?

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15. Suppose a 5-meter diameter telescope were constructed on the dark side of the moon. The viewing there (except for brief periods of sunlight) would be excellent. As an example, what would be the separation between two objects that could just be resolved on the planet Mars in 500 nm light? [The distance to Mars at closest approach is 50 million miles.]
16. What is the energy in eV of a photon of yellow light? $\lambda = 500 \text{ nm}$.