科目：微積分【機電系二年級】

填空題(10題，每題10分，共100分，答錯不倒扣)請將正確化簡答案填寫於答案卷。

1. Find $\frac{dy}{dx}$ where $y = \int_0^{x^2} f(t)\,dt$. (1)

2. At what points on the curve γ given by the equation $y = x^3 - 3x^2 + 2$ is the tangent line l parallel to the line $y = 9x + 4$? (2)

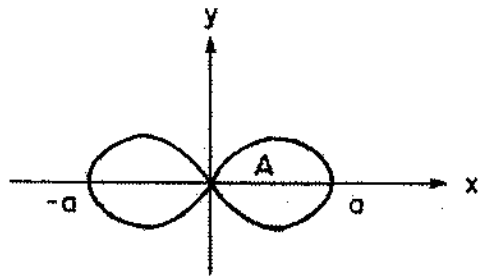
3. Determine $I = \int \frac{x^3+2x^2+x+1}{x^2+2x}\,dx$. (3)

4. Evaluate $I = \int_{-2}^{3} |1 - x^2|\,dx$. (4)

5. Calculate the area A between the curves $y(x) = x^4 + x^3 + 16x - 4$ and $z(x) = x^4 + 6x^2 + 8x - 4$. (5)

6. Find $\frac{d^2y}{dx^2}$ if $x = t^4 + t,\ y = t^3 - t$. (6)

7. Find the area A of the region interior to the right branch ($x \geq 0$) of the curve described by the equation $(x^2 + y^2)^2 = a^2(x^2 - y^2),\ a > 0$. (7)



8. Determine the interval of convergence for the series (8)

$$S(x) = \sum_{k=1}^{\infty} \frac{(x-4)^k}{\sqrt{3^k}}.$$

9. Evaluate $L = \lim_{x\to 0}(1 + 3x)^{(1+5x)/x}$. (9)

10. Find the average of the function $f(x) = \int_x^1 \cos(t^2)\,dt$ on the interval $[0, 1]$. (10)

～全卷完～

科目：普通物理【機電系二年級】

一、單選題，共二十題，每題三分

1. Two objects, X and Y, are held at rest on a horizontal frictionless surface and a spring is compressed between them. The mass of X is 2/5 times the mass of Y. when the spring returns to its original length, X has a kinetic energy of 50 J and Y has a kinetic energy of:
(A) 20 J (B) 8 J (C) 125 J (D) 310 J (E) 50 J.

2. A thin-walled hollow tube rolls without sliding along the floor. The ratio of its translational kinetic energy to its rotational kinetic energy (about an axis through its center of mass) is:
(A) 1 (B) 2 (C) 3 (D) 1/2 (E) 1/3.

3. A solid wheel with mass M, radius R, and rotational inertia $MR^2/2$, rolls without sliding on a horizontal surface. A horizontal force F is applied to the axle and the center of mass has an acceleration a. The magnitudes of the applied force F and the frictional force f of the surface, respectively, are:
(A) F = Ma, f = 0 (B) F = Ma, f = Ma/2 (C) F = 2Ma, f = Ma
(D) F = 2Ma, f = Ma/2 (E) F = 3Ma/2, f = Ma/2.

4. A certain wire stretches 0.90 cm when outward forces with magnitude F are applied to each end. The same forces are applied to a wire of the same material but with three times the diameter and three times the length. The second wire stretches:
(A) 0.10 cm (B) 0.30 cm (C) 0.90 cm (D) 2.7 cm (E) 8.1cm.

5. A spherical shell has inner radius R_1, outer radius R_2, and mass M, distributed uniformly throughout the shell. The magnitude of the gravitational force exerted on the shell by a point particle of mass m located a distance d from the center, outside the inner radius and inside the outer radius, is:
(A) 0 (B) GMm/d^2 (C) $GMm/(R_2^3-R_1^3)$
(D) $GMm(d^3-R_1^3)/d^2(R_2^3-R_1^3)$ (E) $GMm/(d^3-R_1^3)$.

6. A simple pendulum is suspended from the ceiling of an elevator. The elevator is accelerating upwards with acceleration a. The period of this pendulum, in terms of its length L, g, and a is:

(A) $2\pi\sqrt{(L/g)}$ (B) $2\pi\sqrt{(L/(g+a))}$ (C) $2\pi\sqrt{(L/(g-a))}$

(D) $2\pi\sqrt{(L/a)}$ (E) $(1/2\pi)\sqrt{(g/L)}$.

7. If y = 0.02 sin (30x - 400t) (SI units) and if the mass density of the string on which the wave propagates is 0.005 kg/m, then the transmitted power is:
(A) 1.03 W (B) 2.13 W (C) 4.84 W (D) 5.54 W (E) 106 W.

8. The intensity from a sound source is I. If another identical but independent sound source is placed next to it,what will be the change of intensity level?
(A) 2dB (B) 3dB (C) 4dB (D) 5dB (E) 6dB.

9. An ideal gas is allowed to expand adiabatically. Assume the process is reversible. The change in entropy is:
(A) 0 (B) nR ln (V_2/V_1) (C) nR ln (T_2/T_1) (D) kn ln (V_2/V_1)
(E) kn ln (T_2/T_1).

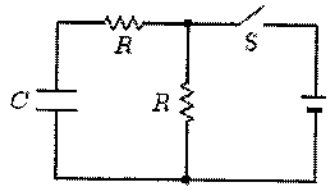
10. A solid conducting sphere (radius = 5.0 cm) has a charge of 0.25 nC distributed uniformly on its surface. If point A is located at the center of the sphere and point B is 15 cm from the center, what is the magnitude of the electric potential difference between these two points?
(A) 23 V (B) 30 V (C) 15 V (D) 45 V (E) 60 V.

11. Two conducting spheres have radii R_1 and R_2, with R_1 greater than R_2. If they are far apart the capacitance is proportional to:
(A) $R_1 R_2/(R_1-R_2)$ (B) $R_1^2-R_2^2$ (C) $(R_1-R_2)/R_1 R_2$ (D) $R_1^2+R_2^2$
(E) none of these.

12. A 30-μF capacitor is charged to 40 V and then connected across an initially uncharged 20-μF capacitor. What is the final potential difference across the 30-μF capacitor?
(a) 15 V (b) 24 V (c) 18 V (d) 21 V (e) 40 V

13. A conductor of radius r, length l and resistivity , has resistance R. It is melted down and formed into a new conductor, also cylindrical, with one fourth the length of the original conductor. The resistance of the new conductor is:
(A) R/16 (B) R/4 (C) R (D) 4R (E) 16R.

14. In the circuit shown, both resistors have the same value R. Suppose switch S is initially closed. When it is then opened, the circuit has a time constant τ_a. Conversely, suppose S is initially open. When it is then closed, the circuit has a time constant τ_b. The ratio τ_a/τ_b is: (A) 1 (B) 2 (C) 0.5 (D) 0.667 (E) 1.5.



15. A loop of wire carrying a current of 2.0 A is in the shape of a right triangle with two equal sides, each 15 cm long. A 0.7 T uniform magnetic field is in the plane of the triangle and is perpendicular to the hypotenuse. The magnetic force on either of the two equal sides has a magnitude of:

(A) zero (B) 0.105 N (C) 0.15 N (D) 0.21 N (E)0.25N.

16. Two long parallel wires are separated by 2.0 cm. The current in one of the wires is three times the other current. If the magnitude of the force on a 2.0-m length of one of the wires is equal to 60 μN, what is the greater of the two currents?
(A) 2.0 A (B) 1.0 A (C) 3.0 A (D) 9.0 A (E) 1.5 A.

17. A 0.20 cm radius cylinder, 3.0 cm long, is wrapped with wire to form an inductor. At the instant the magnetic field in the interior is 5.0 mT the energy stored in the field is about:
(A) 0 (B) 3.8×10^{-6} J (C) 7.5×10^{-6} J (D) 7.5×10^{-4} J (E) 9.9J.

18. Faraday's law states that an induced emf is proportional to:
(A) the rate of change of the magnetic field.
(B) the rate of change of the electric field.
(C) the rate of change of the magnetic flux.
(D) the rate of change of the electric flux.
(E) zero.

19. If an electron travels with speed **v** around a circle of radius **r**, then the magnitude of the orbital magnetic dipole moment is :
(A) evr/2 (B) ev/r (C) ev/2πr (D) 2πer/v (E) 2πev/r.

20. An LC circuit consists of a 1-μF capacitor and a 4 mH inductor. Its oscillation frequency is approximately:
(A) 0.025 Hz (B) 25 Hz (C) 60 Hz (D) 2500 Hz (E) 15800 Hz.

二、計算題，每題二十分

1. A thick spherical shell of inner radius a and outer radius b carries a charge density given by $\rho = \dfrac{ce^{-\frac{r}{a}}}{r^2}$, where a and c are constants. Find expressions of the electric field strength for (a) $r < a$ (b) $a < r < b$ (c) $r > b$.

2. A long coaxial cable consists of two thin-walled concentric conducting cylinders with radii a and b. The inner cylinder carries a steady current i, and the outer cylinder provides the return path for that current. The current sets up a magnetic field between the two cylinders. Calculate the energy stored in the magnetic field for a length l of the cable.