國立中山大學97學年度轉學生招生考試試題

科目：工程數學【機電系三年級】

1. (12%) Find an integrating factor and solve the initial value problem.

$$2\sin(y^2)dx + xy\cos(y^2)dy = 0, \qquad y(2) = \sqrt{\pi/2}$$

2. (12%) Solve the following initial value problem.

$$y'' + 2y' + 101y = 10.4e^x, \qquad y(0) = 1.1, \quad y'(0) = -0.9$$

3. (12%) Solve the non-homogeneous Euler-Cauchy equation.

$$x^3y''' - 3x^2y'' + 6xy' - 6y = x^4 \ln x$$

4. (12%) Find the Laplace transform.

$$te^{-t}\cos\ t$$

5. (16%) Find a basis of eigenvectors and diagonalize the following matrix.

$$\begin{bmatrix} -2.5 & -3 & 3 \\ -4.5 & -4 & 6 \\ -6 & -6 & 8 \end{bmatrix}$$

6. (12%) Find the directional derivative of f at P in the direction of $\mathbf{a}$, where

$$f = x^2 + 3y^2 + 4z^2, \quad P:(1,0,1), \quad \mathbf{a} = -\mathbf{i} - \mathbf{j} + \mathbf{k}$$

7. (12%) Find the unit normal vector for the following surface at the given point.

$$x^2 + y^2 + 2z^2 = 26, \quad P:(2,2,3)$$

8. (12%) Evaluate the following integral. (Hint: the form under integral sign is exact)

$$\int_{(0,\pi)}^{(3,\pi/2)} e^x(\cos y dx - \sin y dy)$$

1. The bent rod in Figure 1 is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by mean of cable BC. Using only ***one equilibrium equation***, obtain a direct solution for the tension in cable BC. The bearing at A is capable of exerting force components only in the z and y directions since it is properly aligned on the shaft. (25%)
2. The hoist supports the 125-kg engine shown in Figure 2. Determine the force in member *DB* and in the hydraulic cylinder *H* of member *FB*. (25%)
3. The mechanism shown in Figure 3 rests on a horizontal surface and is subjected to a horizontal force *P*. The coefficient of sliding friction between the frame *C* and the horizontal surface is 0.20. Frame *C* weighs 108 N. The bar *AB* is 1.2 m long and weighs 27 N. It is attached to the frame *C* by a frictionless pin at *A*. The horizontal spring (of negligible mass) is attached to the midpoint of the bar and exerts a 45 N force on the bar. The force *P* is increased until the contact force between the bar and the frame at point *B* is zero. Determine *P* for this instant. (25%)
4. Figure 4 is a schematic diagram of a disk-brake system. The disk is rigidly attached to the shaft. The shaft, which rotates freely on its bearings, is brought up to an angular speed of 400 rpm. To bring it to stop, equal pad forces $P = 200$ N are applied as shown. The moment of inertia of the disk-brake system is $I_o = 1.00$ kg·m^2, and the coefficient of kinetic friction between the brake pads and disk is $\mu_k = 0.2$. Determine the number of revolutions the shaft undergoes before it comes to rest. Ignore friction of the bearings. (25%)

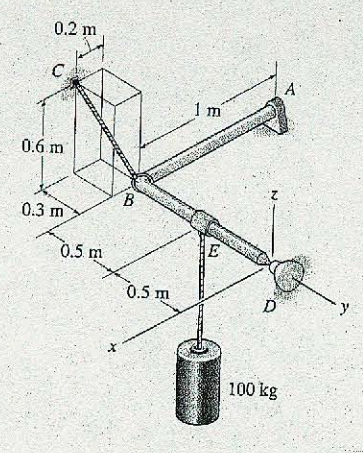

Fig. 1

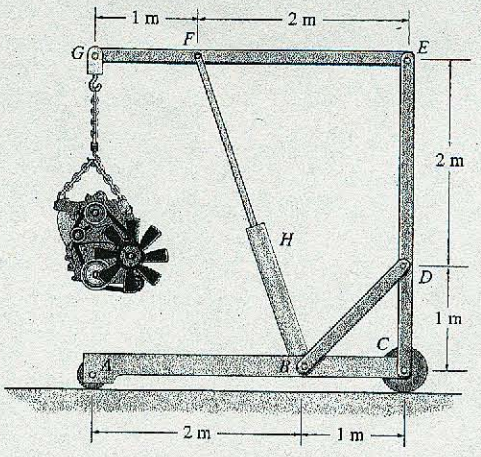

Fig. 2

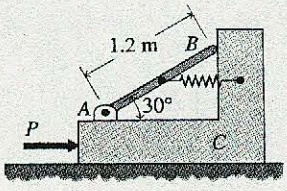

Fig. 3.

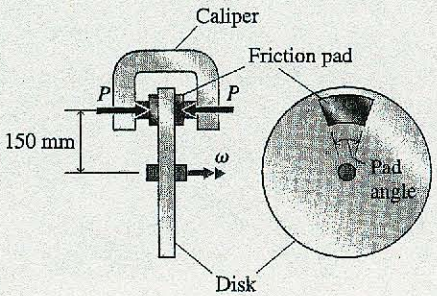

Fig. 4