Exam of Calculus for Departments of Engineering - Electrics

Full marks are 100; the marks are indicated within questions.

I. (15) Give the definition of the indefinite integral, and describe the relation between it and the definite integral (i.e., the fundamental theorem of Calculus).

II. (15) Find the limit

$$\lim_{x\to\infty}\sqrt{x+\sqrt{x}}-\sqrt{x}).$$

III. (15) Let the function

$$f(x)=\frac{(1+x)(2+x)\cdots(9+x)}{(1-x)(2-x)\cdots(9-x)}.$$

Find the derivative $f'(0)$.

IV. (15) Find the limit

$$\lim_{h\to 0}\frac{1}{h}\int_x^{x+h}\frac{du}{u+\sqrt{u^2+1}}.$$

V. (20) Find indefinite integral

$$\int\frac{xe^x}{\sqrt{e^x-1}}dx.$$

VI (20) Determine whether the following singular definite integral is convergent or not,

$$\int_e^{10}\frac{dx}{x\ln x\ln(\ln x)}.$$

1. In Fig.1, two uniform disks have mass m and $3m$ and equal radii R. They are mounted as shown on the same vertical axis with frictionless bearings. The upper disk is given an initial angular velocity ω_0 and then allowed to fall on to the lower disk, which is initially at rest. Friction between the disk surfaces causes them to rotate together with a common rotating speed ω. In terms of given symbols, find a)the final angular speed ω and b)the total amount of frictional thermal energy generated.(12%)

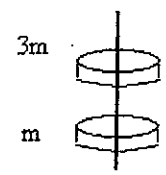

Fig.1

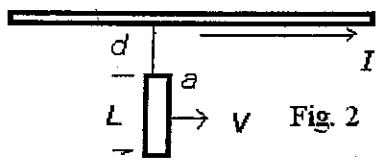

Fig. 2

2. A 30.0 kg projectile is fired at an angle of 60.0 degree above the horizontal and with a velocity of magnitude 200m/s. At the highest point of its trajectory the projectile explodes into two fragments with equal mass, one of which falls vertically with 1.0m/s speed. How far from the point of firing does the other fragment strike the ground? (12%)

3. A 3.0g bullet is fired horizontally to two blocks on a frictionless table. The bullet penetrates the first 1.5kg block and leaves with speed 0.6m/s. And then it strikes the second 2.1kg block and stays in the block. The final speed of the second block is 1.3m/s. Find the initial speed of the bullet. (13%)

4. The long, straight wire in Fig.2 carries constant current I. A metal rod with length L is moving at constant velocity V. Point a is a distance d from the wire. Find the *emf* induced in the rod. (12%)

5. Light of wavelength 440 nm passes through a double slit, yielding a diffraction pattern whose graph of intensity I versus deflection angle θ is shown Fig.3. Calculate a) the slit width and b) the slit separation. (12%)

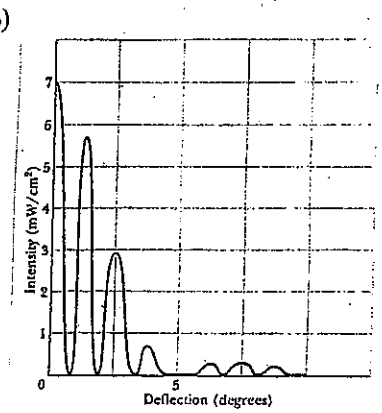

Fig. 3

6. A wave traveling along a string of density 16g/m is described by the wave equation $y(x,t) = 2.5\sin[(7.5m^{-1}x - (110rad/s)t]cm$. Find a) the tension on the string, b) the power transmitted by the wave. (12%)

7. One mole of monatomic ideal gas under the processes in Fig.4 where $V_2 = 3.0V_1$ and 1->2 isothermal, 2->3 constant volume, and 3->1 adiabatic. Find a) $P_2, P_3, and T_3$ b) $W, Q, and \Delta S$ in three processes respectively. All quantities are written in terms of $P_1, V_1,$ and gas constant R. (15%)

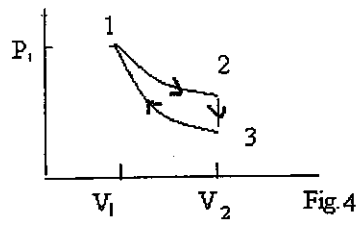

Fig.4

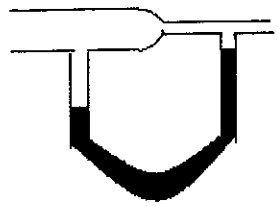
Fig. 5

8. The horizontal section of pipe shown in Fig.5 has a cross-section area of 40.0 cm^2 at the wider portion and 10.0 cm^2 at the constriction. Water is flowing in the pipe, and the discharge from the pipe is $4.00\times10^{-3}m^3/s$. Find the difference in height between the mercury columns in the U-shape tube. (ρ=$13.6\times10^{-3}kg/m^3$) (12%)