

# 國立中山大學 103 學年度轉學考招生考試試題

科目名稱：微積分【機電系二年級】

題號：738001

※本科目依簡章規定「不可以」使用計算機，「不可以」於試題紙上作答

共 1 頁第 1 頁

1.(10 points) Calculate  $\int \frac{4}{\sqrt{1+e^x}} dx$

2.(20 points) Test this series for (a) absolute convergence (10 points) (b) conditional convergence (10 points)

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

3.(10 points) Determine whether or not  $f(x, y)$  has a limit at (1,1)

$$f(x, y) = \frac{x - y^4}{x^3 - y^4}$$

4.(20 points) Let  $\Omega$  be the first-quadrant region bounded by the curves:

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 16, \quad x^2 - y^2 = 1, \quad x^2 - y^2 = 4$$

Evaluate

$$\iint_{\Omega} \frac{1}{4} x^3 y dx dy$$

5.(20 points) Evaluate the line integral

$$\oint_C (1 + 20xy + y^2) dx + (5x^2) dy$$

where  $C$  is the oriented counter clockwise square with vertices (0,0) (1,0) (1,1) (0,1)

6.(20 points) Calculate the flux of the vector field  $v(x, y, z) = xi + j + zk$  across the sphere  $S: x^2 + y^2 + z^2 = 4$  in the outward direction.

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科目名稱：普通物理【機電系二年級】

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共 2 頁第 1 頁

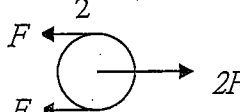
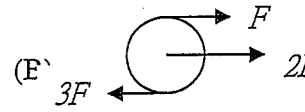
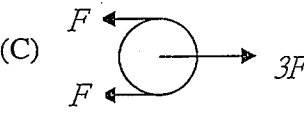
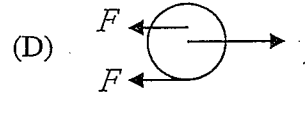
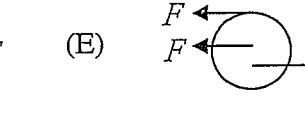
單選題，共 20 題，每題 5 分，總分 100 分，不作答 0 分，答錯倒扣 1 分。

1. Four particles move on a horizontal plane with the same constant acceleration  $a\hat{y}$ , where  $a > 0$ . At  $t = 0$ , their initial velocity vectors are (a)  $12\hat{x} + 10\hat{y}$ , (b)  $12\hat{x} + 8\hat{y}$ , (c)  $8\hat{x} + 12\hat{y}$ , (d)  $12\hat{x} + 12\hat{y}$ . If the respective displacements at  $t > 0$  are  $R_a, R_b, R_c, R_d$ , which of the following is correct? (A)  $R_a > R_b > R_c > R_d$ , (B)  $R_c > R_b > R_d > R_a$ , (C)  $R_d > R_a > R_b > R_c$ , (D)  $R_d > R_a > R_c > R_b$ , (E)  $R_a > R_c > R_b > R_d$ .

2. Given the position vector  $\vec{r} = (t^2 + 2)\hat{i} + (3t - 5)\hat{j} + (-t^2 - t + 1)\hat{k}$ , what is the magnitude of acceleration at  $t = 1$ ? (A)  $\sqrt{5}$ , (B)  $\sqrt{14}$ , (C)  $2\sqrt{2}$ , (D) 2, (E) 0.

3. What is the rotational inertia of a solid sphere of radius  $R$  and mass  $M$  about an axis passing through the center of the sphere? (A)  $\frac{1}{2}MR^2$ , (B)  $\frac{2}{3}MR^2$ , (C)  $\frac{1}{4}MR^2$ , (D)  $\frac{2}{5}MR^2$ , (E)  $MR^2$ .

4. A man of mass  $m$ , initially at rest, falls from a point at  $(d, 0, 0)$ . If the gravitational acceleration is  $\vec{a}_g = -g\hat{z}$ , find the angular momentum of the falling man about the origin at time  $t$ : (A)  $dmgt\hat{x}$ , (B)  $\frac{1}{2}dmgt\hat{x}$ , (C)  $dmgt\hat{y}$ , (D)  $\frac{1}{2}dmgt\hat{y}$ , (E)  $-\frac{1}{2}dmgt\hat{z}$ .

5. Which wheel is at equilibrium? (A)  (B)  (C)  (D)  (E) 

6. Imagine that a tunnel is drilled from the north pole through the center of the Earth to reach the south pole, and a particle of mass  $m$  is dropped from one end of the tunnel what is the period of oscillation?

(A)  $\sqrt{\frac{3\pi}{G\rho}}$ , (B)  $\sqrt{\frac{3\pi}{4G\rho}}$ , (C)  $\sqrt{\frac{3}{4\pi G\rho}}$ , (D)  $\sqrt{\frac{3\pi}{2G\rho}}$ , (E)  $\sqrt{\frac{4\pi}{G\rho}}$ .

7. A linear SHM takes place at a frequency of 2 Hz about  $x=0$ . At  $t=0$ , the displacement is  $x(0)=3$  and the velocity is  $v(0)=0$ . What is  $v(1)$ ? (A) 0, (B)  $\pi$ , (C)  $2\pi$ , (D)  $3\pi$ , (E)  $6\pi$ .

8. If the speed and frequency of a sound wave are  $v$  and  $f$  when measured by a detector at rest with respect to the source, what is the frequency measured by the detector moving at a speed of  $v_d$  while the

sound source is moving in parallel at  $v_s$  with respect the rest frame? (A)  $\frac{v+v_d}{v+v_s}f$ , (B)  $\frac{v-v_d}{v-v_s}f$ ,

(C)  $\frac{v+v_s}{v+v_d}f$ , (D)  $\frac{v-v_d}{v+v_s}f$ , (E)  $\frac{v+v_s}{v-v_d}f$ .

9. Find the heat capacity of two metals, each of masses  $M_1$  and  $M_2$  and specific heat of  $X_1$  and  $X_2$ , when

they are connected: (A)  $\frac{X_1}{M_1} + \frac{X_2}{M_2}$ , (B)  $\frac{M_1}{X_1} + \frac{M_2}{X_2}$ , (C)  $M_1X_1 + M_2X_2$ , (D)  $\frac{X_1 + X_2}{M_1 + M_2}$ , (E)  $\frac{M_1 + M_2}{X_1 + X_2}$ .

背面有題

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共 2 頁第 2 頁

10. A particle of temperature  $T$  is located in an environment of temperature  $T_{env}$ . What is the temperature dependence of thermal radiation energy that is emitted from the particle? (A)  $T^4$ , (B)  $T_{env}^4$ , (C)  $T^4 - T_{env}^4$ , (D)  $T$ , (E)  $T_{env}$ .
11. The speed of molecules in an ideal gas follows Maxwell's distribution law. Which of the following is correct when comparing the average speed  $v_a$ , most probable speed  $v_p$ , and root mean square speed  $v_{rms}$ ? (A)  $v_p > v_a > v_{rms}$ , (B)  $v_{rms} > v_a > v_p$ , (C)  $v_{rms} > v_p > v_a$ , (D)  $v_a > v_{rms} > v_p$ , (E)  $v_p > v_{rms} > v_a$ .
12. A point charge  $Q$  is located at a distance of  $R/2$  from the center of a spherical shell of radius  $R$ . If the shell is electrically neutral, what is the net flux of electric field through the sphere shell? (A)  $Q$ , (B)  $\epsilon_0 Q$ , (C)  $Q/\epsilon_0$ , (D) 0, (E)  $Q/2$ .
13. Three point charges held at three corners of an equilateral triangle with side  $d$ . If  $Q_1 = +q$ ,  $Q_2 = +q$ ,  $Q_3 = -4q$ , the total electrostatic energy is (A)  $\frac{6q^2}{4\pi\epsilon_0 d}$ , (B)  $\frac{-6q^2}{4\pi\epsilon_0 d}$ , (C)  $\frac{7q^2}{4\pi\epsilon_0 d}$ , (D)  $\frac{-7q^2}{4\pi\epsilon_0 d}$ , (E) 0.
14. An RC circuit consists of a resistor, a capacitor, and an ideal battery of emf  $\epsilon$  in series. What is the charge in the capacitor at time  $t = RC$  after it starts charging? (A)  $0.37C\epsilon$ , (B)  $0.5C\epsilon$ , (C)  $0.63C\epsilon$ , (D)  $C\epsilon$ , (E) 0. (Note:  $e = 2.7183$  and  $1/e = 0.3679$ )
15. A long straight wire of radius  $R$  carries a uniformly distributed current  $i$ . What is the magnetic field at a distance  $r < R$  from the center of the wire? (A) 0, (B)  $\frac{\mu_0 i}{2\pi r}$ , (C)  $\frac{\mu_0 i}{2\pi R}$ , (D)  $\frac{\mu_0 i r}{2\pi R^2}$ , (E)  $\frac{\mu_0 i R}{2\pi r^2}$ .
16. Which of what follows describes the Faraday's law? (A)  $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ , (B)  $\oint \vec{B} \cdot d\vec{s} = \mu_0 i$ , (C)  $\oint \vec{B} \cdot d\vec{A} = 0$ , (D)  $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ , (E)  $\oint \vec{E} \cdot d\vec{A} = 0$ .
17. Consider a long solenoid of  $n$  turns per unit length carrying in it a current  $i$ . What is the stored energy density of this solenoid? (A) 0, (B)  $\frac{1}{2} \mu_0 n^2 i^2$ , (C)  $\frac{1}{2} \mu_0 n i^2$ , (D)  $\mu_0 n^2 i^2$ , (E)  $\mu_0 n i^2$ .
18. What is the angular frequency of resonance for a series RLC circuit? (A)  $\sqrt{RC}$ , (B)  $\sqrt{LC}$ , (C)  $\frac{1}{\sqrt{RC}}$ , (D)  $\frac{1}{\sqrt{LC}}$ , (E)  $\sqrt{\frac{R}{LC}}$ .
19. An unpolarized light is incident from material 1 with refractive index  $n_1$  into material 2 with index  $n_2$ . What is the angle of incidence with respect to the interface when the reflected light becomes fully polarized? (A)  $\cos^{-1} \frac{n_2}{n_1}$ , (B)  $\sin^{-1} \frac{n_1}{n_2}$ , (C)  $\cos^{-1} \frac{n_1}{n_2}$ , (D)  $\tan^{-1} \frac{n_1}{n_2}$ , (E)  $\tan^{-1} \frac{n_2}{n_1}$ .
20. A single-slit, cut from a thin plate to a width  $a$ , is illuminated by a plane wave of light with wavelength  $\lambda$  at normal incidence onto the plate. Find  $a$  if the first minimum appear at  $\theta$ . (A)  $\frac{1}{2} \lambda \sin \theta$ , (B)  $\lambda \sin \theta$ , (C)  $\frac{\lambda}{2 \sin \theta}$ , (D)  $\frac{\lambda}{\sin \theta}$ , (E)  $\frac{3\lambda}{2 \sin \theta}$ .