

Calculus

1. (15 points) Let $1 + e^{xy} = y^2 + x$. Find $\frac{d^2y}{dx^2}$ in terms of x and y .

2. (15 points) Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{1 + \dots + n}$$

3. (15 points) Evaluate

$$\int \frac{x^2 + 3x + 4}{x^2 + 2x + 2} dx$$

4. (15 points) Let $F(x, y, z) = x^2 - y^2z$. What is the direction of greatest increases for the function F at the point $(1, 2, 3)$?

5. (20 points) Use the method of Lagrange multipliers to find the point on the plane $x + 2y + 3z = 6$ that is nearest to the origin.

6. (20 points) Evaluate the integral

$$\int_C -xydx + x^2dy,$$

where C is the the right triangle with vertices $(0, 0)$, $(0, 2)$, and $(1, 0)$.

國立中山大學 102 學年度轉學考招生考試試題

科目名稱：普通物理【機電系二年級】

題號：738002

※本科目依簡章規定「不可以」使用計算機

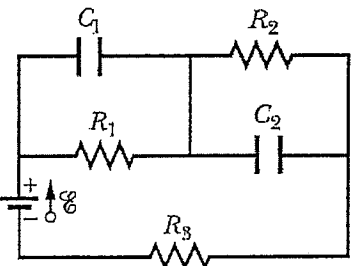
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◎請在答案卷上標明選擇題題號與答案如下：

1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ 7. _____ 8. _____ 9. _____ 10. _____ 11. _____
12. _____ 13. _____ 14. _____ 15. _____ 16. _____ 17. _____

一、單選題(每題 5 分，不倒扣)：

- With a mass of 4 kg, a particle is acted by F_1 and F_2 to accelerate at $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ (m/s^2). If $F_1 = 2\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$ (N), then $F_2 =$ (A) $2\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$, (B) $2\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$, (C) $F_2 = 6\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$, (D) $F_2 = 6\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$, (E) $F_2 = 2\mathbf{i} - 5\mathbf{j}$ (N).
- A particle moves along the x -axis according to the equation $7t = x^2 + 8x + 5$, where x is the position in meter and t is the time in second, find its speed when $x = 3.0$ m.
(A) 2.0, (B) 1.5, (C) 1, (D) 0.5, (E) 0.1 ms^{-1} .
- Which in the following does *not* represent the dimension of force?
(A) ma , (B) kx , (C) qE , (D) qB , (E) mv^2/x , (m : mass, x : position, v : speed, a : acceleration, k : force constant, q : charge, E : electric field, B : magnetic induction).
- A solid wheel with mass M , radius R , rolls *without sliding* on a horizontal surface and its rotational inertia is $MR^2/2$. If the center of mass is accelerating at a . Find the applied force F acting on the axle and the frictional force f on the wheel surface. (F, f) =
(A) $(Ma, 0)$, (B) $(Ma, Ma/2)$, (C) $(2Ma, Ma)$, (D) $(2Ma, Ma/2)$, (E) $(3Ma/2, Ma/2)$.
- A wheel takes 3.00 s to rotate 36.0 rounds. Its angular speed at the end of the 3 s interval is 27π rad/s. What is the constant angular acceleration of the wheel?
(A) 2.0, (B) 1.0, (C) 3.0, (D) 0.5, (E) $4.0 \text{ (rad}\cdot\text{s}^2)$.
- A charge of 10 C is first put on a spherical conducting shell and then another point charge of -3 C is put at the center. Find the net charge on the outer surface of the shell:
(A) -7 C, (B) -3 C, (C) 0 C, (D) $+3$ C, (E) $+7$ C.
- A capacitor of capacitance C and an inductor of inductance L are connected on both ends, the resonance angular frequency is:
(A) LC , (B) $(LC)^{1/2}$, (C) $(LC)^2$, (D) $(1/LC)^{1/2}$, (E) L/C .
- A magnetic field B passing in perpendicular through a square wire loop of area A . The magnetic flux through the loop is: (A) 0, (B) $BA/2$, (C) BA , (D) $2BA$, (E) $3BA$.
- In the right figure, $R_1 = 5 \Omega$, $R_2 = 10 \Omega$, $R_3 = 15 \Omega$, $C_1 = 5 \mu\text{F}$, $C_2 = 10 \mu\text{F}$ and the ideal battery has an emf $\mathcal{E} = 20$ V. In steady state, the total energy stored in the two capacitors is: (A) 2.78×10^{-5} , (B) 1.12×10^{-4} , (C) 2.22×10^{-4} , (D) 2.50×10^{-4} , (E) 4.72×10^{-4} (J).
- A monochromatic light ($\lambda = 560$ nm) is incident on a thin film with refractive index $n = 1.40$. How thick must the film be in order for destructive interference to occur when reflected? (A) 100, (B) 200, (C) 150, (D) 250, (E) 50 (nm).
- Assume the pupil diameter is 0.50 cm and $n = 1.22$ for the naked eye. What is the maximal distance if one is to distinguish between two blue lights ($\lambda = 500$ nm) separated by 1.5 m? (A) 6.0, (B) 12, (C) 9.0, (D) 3.0, (E) 15 (km).
- For diffraction of waves by a single slit of width a at a distance D from the screen, which is right?
(A) the first maximum occurs at $a \sin \theta = \lambda$, (B) smaller a will result in larger separations between minima, (C) the intensity of each maximum is the same, (D) the diffraction is more easily observed for lights of shorter wavelength than those of longer wavelength, (E) the first maximum occurs at $a \sin \theta = 3\lambda/2$.



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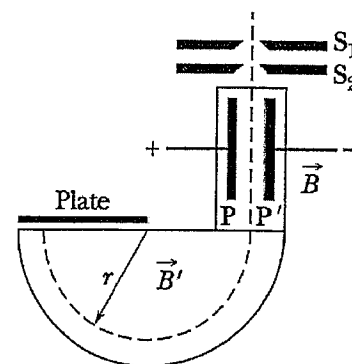
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二、複選題(每題 6 分，答錯倒扣)：

13. Accelerated by a potential difference V , two charged particles of the same charge q but different mass m_1 and m_2 enter a uniform field B' following a circular path illustrate in the right figure. What in the following are correct:



(A) The kinetic energy is qVB' for both particles, (B) the speed of m_1 is $(2qV/m_1)^{1/2}$, (C) the radius of the path in B' for m_2 is $(m_2V/qB'^2)^{1/2}$, (D) $r_1/r_2 = (m_2/m_1)^{1/2}$, (E) $r_1/r_2 = (m_1/m_2)^{1/2}$.

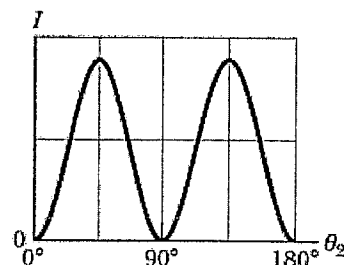
14. A solid sphere of radius R has a volume charge density $\rho = \alpha r/R$, where r is the radial distance from the sphere's center. What's correct?

(A) The Gauss's Theorem is not applicable, (B) the total charges on the sphere is $\alpha\pi R^3$, (C) the magnitude of the electric field at $r = 0$ is zero, (D) the magnitude of the electric field at $r = \frac{R}{2}$ is $\frac{1}{16} \frac{\alpha R}{\epsilon_0}$, (E) the magnitude of the electric field at $r = R$ is $\frac{\alpha R}{\epsilon_0}$.

15. Which of the following are correct?

(A) There can be no electric field inside a solid conducting sphere, (B) the Gauss's Theorem of magnetism suggests that no magnetic monopole exists, (C) electric current flowing in a wire will produce a magnetic field, (D) a static magnetic field threaded through a loop will cause an emf, (E) alternately changing electric and magnetic fields will generate an electromagnetic wave.

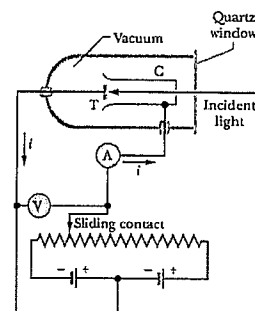
16. A beam of unpolarized light is sent into a sequence of three polarizing sheets. The polarization angles are fixed at θ_1 and θ_3 for sheet 1 and 3, respectively, but for sheet 2 the angle θ_2 is variable. The transmitted intensity of the light as a function of θ_2 is shown to the right.



Select the correct answers from below. (A) $\theta_1 = 90^\circ$ or $\theta_3 = 90^\circ$, (B) transmission rate of the unpolarized light after sheet 1 is $1/2$, (C) $|\theta_1 - \theta_3| = 90^\circ$, (D) the transmitted intensity is 0 if one takes out the sheet 2, (E) transmission rate is $1/8$ if $\theta_2 = 30^\circ$.

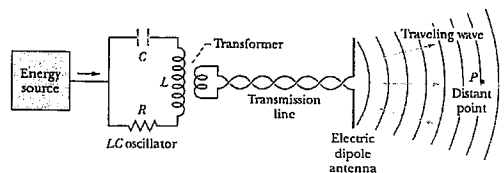
17. A setup to test photoelectric effect is illustrated to the right.

(A) The kinetic energy K_{\max} of the photoelectron is proportional to the stopping potential, (B) for a light of given frequency, K_{\max} does not depend on the intensity of the light source, (C) increasing the intensity of the light source would cause larger K_{\max} , (D) the slope of the "stopping potential vs. incident light frequency" plot depends on the metals, (E) the work function of the target metal can be derived (T is the target, C is the cathode in the diagram).



三、計算題(10 分)：

An oscillator antenna, as shown in the figure below, is connected through a transmission line and a transformer to an LCR oscillator circuit. If R is so negligibly small that one can take $R=0$, what is the (a) speed (4%), (b) frequency (3%), and (c) wavelength of the electromagnetic wave emitted by the antenna if $L = 0.30 \mu\text{H}$ and $C = 30.0 \text{ pF}$ (3%)?



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