

Do all the problems with details.

1. (a) Find $f'(2)$, where $f(x) = \int_x^{x^2} t^2 e^{t^2} dt$. (10%)

(b) Find $\lim_{x \rightarrow \infty} x^2 e^{-x}$ if it exists. (10%)

2. (a) Evaluate

$$\int_1^{\infty} \frac{1}{1+e^x} dx. \quad (10\%)$$

(b) Evaluate the integral $\int_0^{\infty} t^2 e^{-st} dt$ for $s > 0$. (10%)

3. (a) Find the radius of convergence of

$$\sum_{n=0}^{\infty} \binom{n+2}{2} x^n. \quad (10\%)$$

(b) Find the sum of the power series in (a). (10%)

4. (a) Find all critical points for $f(x, y) = x^2 + y^2 + 2x - 4y - 12$ and determine whether each corresponds to a relative maximum, a relative minimum, or a saddle point. (10%)

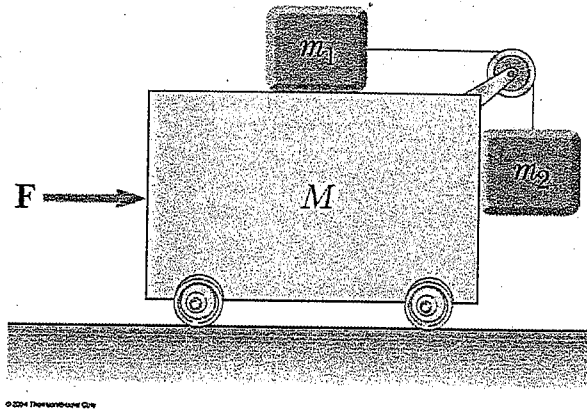
(b) Evaluate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \frac{\sqrt{x^2+y^2}}{1+x^2+y^2} dy dx. \quad (10\%)$$

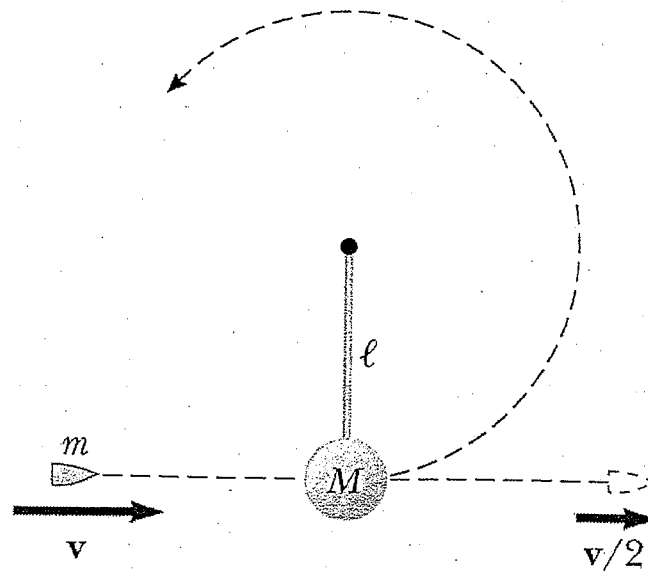
5. (a) Evaluate the line integral $\int_C xy^2 dt$, where C is the curve $x = 2t$, $y = e^t$, $0 \leq t \leq 1$. (10%)

(b) Find the work done by the force field $\mathbf{F}(x, y) = x \mathbf{i} - y^2 \mathbf{j}$ in moving a particle from $(1, 0)$ to $(0, 1)$ along the quarter-circle $\mathbf{r}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j}$, $0 \leq t \leq \pi/2$. (10%)

1. What horizontal force must be applied to the cart shown in the following figure in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (Hint: Note that the force exerted by the string accelerates m_1 .) 10%



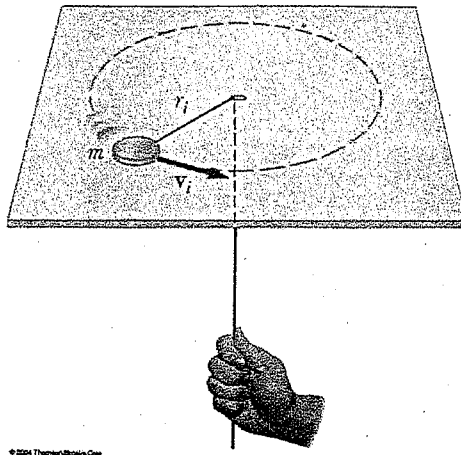
2. As shown in the following figure, a bullet of mass m and speed v passes completely through a pendulum bob of mass M . The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle? 10%



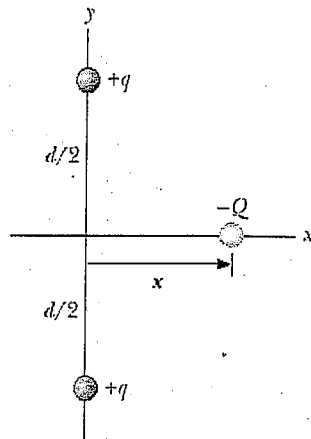
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科目：普通物理【機械與機電工程學系二年級】

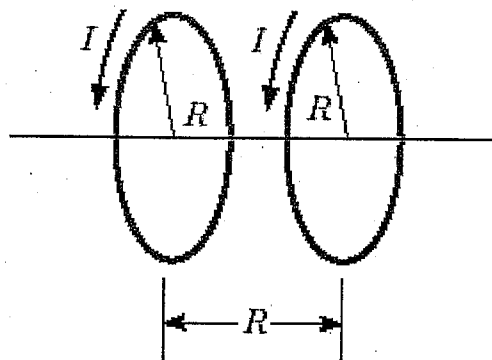
3. A puck of mass m is attached to a cord passing through a small hole in a frictionless, horizontal surface. The puck is initially orbiting with speed v_i in a circle of radius r_i . The cord is then slowly pulled from below, decreasing the radius of the circle to r . (a) What is the speed of the puck when the radius is r ? 3% (b) Find the tension in the cord as a function of r . 3% (c) How much work W is done in moving m from r_i to r ? (Note: The tension depends on r .) 4%



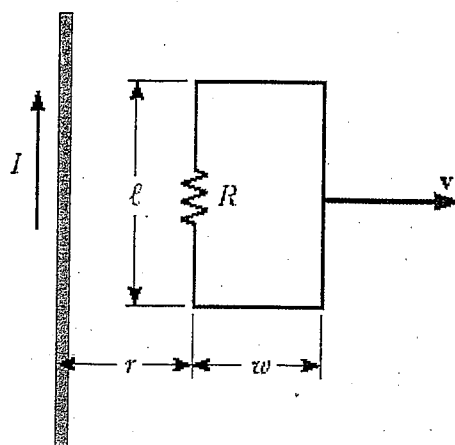
4. One mole of an ideal gas is contained in a cylinder with a movable piston. The initial pressure, volume, and temperature are P_i , V_i , and T_i , respectively. Find the work done on the gas for the following processes and show each process on a PV diagram: (a) An isobaric compression in which the final volume is one-half the initial volume. 3% (b) An isothermal compression in which the final pressure is four times the initial pressure. 4% (c) An isovolumetric process in which the final pressure is three times the initial pressure. 3%
5. Two identical particles, each having charge $+q$, are fixed in space and separated by a distance d . A third point charge $-Q$ is free to move and lies initially at rest on the perpendicular bisector of the two fixed charges a distance x from the midpoint between the two fixed charges as shown in the the following figure. (a) Show that if x is small compared with d , the motion of $-Q$ will be simple harmonic along the perpendicular bisector. Determine the period of that motion. 5% (b) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges, if initially it is released at a distance $a \ll d$ from the midpoint? 5%



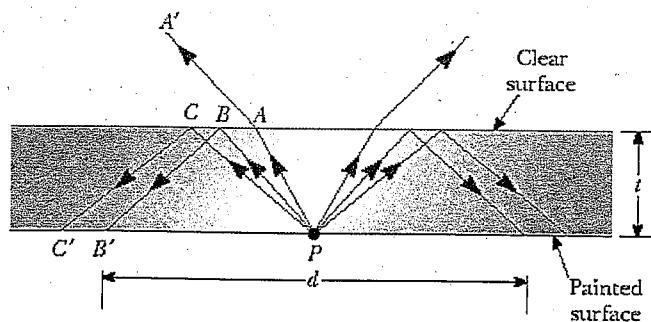
6. Two circular coils of radius R , each with N turns, are perpendicular to a common axis. The coil centers are a distance R apart. Each coil carries a steady current I in the same direction, as shown in the figure. (a) Find the magnetic field on the axis at a distance x from the center of one coil. 5% (b) Show that dB/dx and d^2B/dx^2 are both zero at the point midway between the coils. This means the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called *Helmholtz coils*. 5%



7. A rectangular loop of dimensions ℓ and w moves with a constant velocity v away from a long wire that carries a current I in the plane of the loop. The total resistance of the loop is R . Derive an expression that gives the current in the loop at the instant the near side is a distance r from the wire. 10%



8. A. H. Pfund's method for measuring the index of refraction of glass is illustrated in the following figure. One face of a slab of thickness t is painted white, and a small hole scraped clear at point P serves as a source of diverging rays when the slab is illuminated from below. Ray PBB' strikes the clear surface at the critical angle and is totally reflected, as are rays such as PCC' . Rays such as PAA' emerge from the clear surface. On the painted surface there appears a dark circle of diameter d , surrounded by an illuminated region, or halo. Derive an equation for n in terms of the measured quantities d and t . 10%



9. An oil film ($n = 1.45$) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the color of the light in the visible spectrum most strongly reflected, 5% and (b) the color of the light in the spectrum most strongly transmitted. Explain your reasoning 5%.
10. Light from an argon laser strikes a diffraction grating that has 5 310 grooves per centimeter. The central and firstorder principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the laser light. 10%