科目：微積分【應數系二年級】

填空題(10題，每題10分，共100分，答錯不倒扣)請將正確化簡答案填寫於答案卷。

1. Find $\frac{d^3y}{dx^3}$ when $y = \frac{1+x}{1-x}$. __(1)__

2. Find the equation of the line tangent to the curve whose equation is $x^3 - 4xy + y^3 = 0$, at the point $(2, 2)$. __(2)__

3. Integrate $I = \int \frac{dx}{\sqrt{1+e^x}}$. __(3)__

4. Evaluate $I = \int_0^{\pi/2} \cos^2 3\theta \, d\theta$. __(4)__

5. Evaluate $I = \int_{-1}^{1} \frac{dx}{\sqrt{|x|}}$. __(5)__

6. Find the area A between $y = x^2 - 6x + 8$ and $y = 2x - 7$. __(6)__

7. Find the slope of the cycloid $x = 2(t - \sin t), y = 2(1 - \cos t)$ at the point where $t = \frac{\pi}{2}$. __(7)__

8. Compute the area A of one leaf of the following polar graph __(8)__

$$r = 1 + \sin 2\theta.$$

9. Evaluate $l = \lim_{n\to\infty} \frac{1 + \sqrt[n]{e} + \sqrt[n]{e^2} + \cdots + \sqrt[n]{e^{n-1}}}{n}$. __(9)__

10. If $[x]$ denotes the greatest integer smaller than or equal to x, evaluate the integral

$$\iint_R [x+y] \, dA$$

where $R = \{(x, y) | 1 \le x \le 3,\ 2 \le y \le 5\}$. __(10)__

~全卷完~

科目：線性代數【應數系二年級】

National Sun Yat-sen University
Department of Applied Mathematics
Linear Algebra: Exam
Question Paper

Date: Thursday, July 9, 2009
Mark: 100
Time: 80 minutes
Note: This question paper is composed of four (4) questions. Attempt all of them.

Question One [20 marks]

Given the linear system of equations:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 3x_3 & = & 1 \\ -2x_1 & - & x_2 & + & 2x_3 & = & 0 \\ & & 3x_2 & - & x_3 & = & -2. \end{array} \qquad (*)$$

(1.1) Use the Gauss-Jordan elimination to solve the system (*). [8 marks]

(1.2) Use the LU-decomposition to solve the system (*). [12 marks]

Question Two [30 marks]

(2.1) State the definition of eigenvalues and eigenvectors for a square matrix A. [3 marks]

(2.2) Assume that λ_1 and λ_2 are two distinct eigenvalues of A, and x_1 and x_2 are eigenvectors of A corresponding to λ_1 and λ_2, respectively.

(2.2.1) Prove that x_1 and x_2 are linearly independent. [7 marks]

(2.2.2) Prove that $x_1 + x_2$ is not an eigenvector of A. [7 marks]

(2.3) Find an orthogonal matrix P which diagonalizes the matrix

$$A = \begin{bmatrix} 5 & -2 & -1 \\ -2 & 2 & -2 \\ -1 & -2 & 5 \end{bmatrix}.$$

[13 marks]

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Question Three [30 marks]

(3.1) Prove that the quadratic form

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + x_2x_3$$

is positive definite. [6 marks]

(3.2) Find all values of k so that the quadratic form

$$f(x_1, x_2, x_3) = kx_1^2 + kx_2^2 + kx_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$$

is positive definite. [9 marks]

(3.3) Find an orthogonal matrix Q such that under the substitution $x = Qy$ the quadratic form

$$f(x_1, x_2, x_3) = 5x_1^2 + 2x_2^2 + 5x_3^2 - 4x_1x_2 - 2x_1x_3 - 4x_2x_3$$

is turned into its standard form

$$f(x_1, x_2, x_3) = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2.$$

[10 marks]

(3.4) Find the maximum value of the quadratic form $f(x_1, x_2, x_3)$ in part (3.4) subject to the constraint $x_1^2 + x_2^2 + x_3^2 = 1$, and find a unit vector at which this maximum value is attained. [5 marks]

Question Four [20 marks]

Given the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & -1 \\ 1 & 2 & -2 & 1 & -2 \\ 4 & -1 & 4 & 1 & 1 \end{bmatrix}.$$

(4.1) Find the rank of A. [5 marks]

(4.2) Find a basis for the row space of A which consists entirely of row vectors of A. [5 marks]

(4.3) Express each row vector of A other than the basis row vectors found in (4.2) as a linear combination of the basis row vectors of A found in (4.2). [5 marks]

(4.4) Find the null space of A. [5 marks]

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