國立中山大學97學年度轉學生招生考試試題

科目：微積分【應數系二年級】 共1頁第1頁

共十題，每題10分。答題時，每題都必須寫下題號與詳細步驟。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Find the dimensions of the rectangle of maximum area A that can be inscribed in the portion of the parabola $y^2 = 4px$ intercepted by the line $x = a$ where p and a are positive constants.

2. If $F = 1/r^2$ and F is measured as 4 ± 0.05, estimate r.

3. Evaluate the following limit:

$$\lim_{n\to\infty} \frac{\pi}{6n}\left[\sec^2\left(\frac{\pi}{6n}\right) + \sec^2\left(\frac{2\pi}{6n}\right) + \cdots + \sec^2\left(\frac{(n-1)\pi}{6n}\right) + \frac{4}{3}\right].$$

4. A solid has a base in the form of an ellipse with major axis 10 and minor axis 8. Find its volume if every section perpendicular to the major axis is an isosceles triangle with altitude 6.

5. Find the following integral:

$$\int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2}\, dx.$$

6. Find the following integral:

$$\int \frac{dx}{2 + \cos x}$$

7. Find the Maclaurin series of $\tan^{-1} x$ and evaluate the following sum

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

8. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{\ln(n+1)}$.

9. Find the volume bounded by the paraboloid $x^2 + y^2 = 4z$, the cylinder $x^2 + y^2 = 8y$, and the plane $z = 0$.

10. Find the integral of the function

$$f(x, y, z) = \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4}$$

over the unit ball $B = \{(x, y, z) \mid x^2 + y^2 + x^2 \leq 1\}$.

~全卷完~

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1 (10 points). Transform the basis $\{[1,0,1],[0,1,2],[2,1,0]\}$ for R^3 into an orthonormal basis.

2 (15 points). Suppose A is an orthogonal $n \times n$ matrix. Prove that for any $x \in R^n$, $||Ax|| = ||A^{-1}x||$.

3 (10 points). Find the projective matrix for $W = sp(\mathbf{a}_1, \mathbf{a}_2)$ in R^3, where $\mathbf{a}_1 = [\frac{3}{5}, \frac{4}{5}, 0]$ and $\mathbf{a}_2 = [0,0,1]$.

4 (10 points). Prove that for every positive integer n for every real number a, the set

$$\{1, x-a, (x-2)^2, \cdots, (x-a)^n\}$$

is a basis for the vector space P_n of polynomials of degree at most n.

5 (15 points). Assume A is square matrix all of whose eignevalues are real. Prove that the product of its eigenvalues is $\det(A)$.

6 (15 points). Let $T: V \to V$ be a linear transformation of vector space V into itself. Prove that if $\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_k}$ are eigenvectors of T corresponding to distinct nonzero eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_k$, then the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ is independent.

7 (10 points). Find a Jordan canonical form and a Jordan basis for the following matrix:

$$\begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

8 (15 points) Prove Schur's lemma: Let A be an $n \times n$ matrix. There is a unitary matrix U such that $U^{-1}AU$ is upper triangular.