國立中山大學九十三學年度轉學生招生考試試題

科目：微積分【應數系二年級】

(1) 求下列極限 (20%)

(a) $\lim_{x\to\infty}\left[\cos\left(x+\frac{1}{x}\right)-\cos x\right]$ (b) $\lim_{x\to 0}\frac{x^2\sin\frac{1}{x}}{\tan x}$

(2) 求下列積分 (30%)

(a) $\int \frac{2x+1}{x^2+4}\,dx$ (b) $\int x\sqrt{\sqrt{x}+2}\,dx$ (c) $\int \frac{1}{1+3\cos x}\,dx$

(3) 求由直線 $y=x$ 與函數 $f(x)=x^3-2x^2+2$ 之圖形所圍區域之面積. (15%)

(4) 求橢圓体 $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 之體積. (15%)

(5) 一直徑 8 呎深 10 呎之正圓錐容器 (頂點在下), 每分鐘注入 5 立方呎之水, 試問當水深為 6 呎時, 水面上升之速度為何? (10%)

(6) 求冪級數 $\sum_{n=1}^{\infty}\frac{(2x-3)^n}{2^n\sqrt{n}}$ 之收斂區間. (10%)

國立中山大學九十三學年度轉學生招生考試試題

科目：線性代數【應數系二年級】

Linear Algebra （注意：每個題目需證明或說明清楚，只填答案不計分。）

Let $\mathbb{R}$ be the set of all real numbers.

1. Let $T(x,y,z)=(x+y+z,x+2y+z,x+y+z)$ be a function from $\mathbb{R}^3$ to $\mathbb{R}^3$, $\alpha=\{(1,0,0),(0,1,0),(0,0,1)\}$ and $\beta=\{(1,1,1),(1,1,0),(1,0,0)\}$ be the ordered bases.

(a) Show that T is a linear transformation. (6%)

(b) Find the image and the kernel of T. (6%+6%)

(c) Let $[T]_\alpha^\beta$ be the matrix representation of T with respect to α and β. Find $[T]_\alpha^\beta$. (6%)

(d) Let $[T]_\alpha$ be the matrix representation of T with respect to α. Find $\det([T]_\alpha)$. (6%)

(e) Find the characteristic polynomial of T. (6%)

(f) Is T diagonalizable? (10%)

2. Give a linear system $\begin{cases} x+y+z=a \\ x+2y+3z=b \\ 2x+3y+4z=c \end{cases}$ and $x,y,z\in\mathbb{R}$.

(a) Let $(a,b,c)=(1,2,3)$. Find all solutions of the linear system. (10%)

(b) Prove that (a,b,c) is a linear combination of (2,3,5) and (2,5,7) if and only if the linear system has at least one solution. (12%)

3. Let $\mathbb{R}^3$ be an inner product space with the dot product and $U=\{(x,y,z)\in\mathbb{R}^3: x+2y+3z=0\}$.

(a) Show that U is a linear subspace for $\mathbb{R}^3$. (6%)

(b) Find an orthonormal basis for U. (8%)

4. Determine each following statement either is true or false. If true, prove it; if false, give a counterexample. (6%×3)

(a) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$. Then T is one to one if and only if T is onto.

(b) Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. Then T is diagonalizable.

(c) Suppose u and v are linearly independent vectors of $\mathbb{R}^3$ and w is a vector in $\mathbb{R}^3-\{u,v,(0,0,0)\}$; that is, $w\notin\{u,v,(0,0,0)\}$. Then u,v,w are linearly independent.