

國立中山大學九十一學年度轉學生招生考試試題

科目：微積分【應數系二年級】

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Do all problems in detail.

(1)

(a) [10%] Evaluate $\int_0^\infty e^{-st} \cos at \, dt$ for $a, s > 0$.

(b) [10%] Evaluate $\int_0^2 \int_y^e e^{x^2} \, dx \, dy$.

(2)

(a) [15%] Determine convergency of the series $\sum_{n=1}^\infty \frac{n(n^2+1)}{(n+2)^4}$.

(b) [10%] Find the interval of convergence of the power series $\sum_{n=1}^\infty \frac{(x-1)^n}{(n+1)(-2)^n}$.

(3)

(a) [10%] Find the points on the sphere $x^2 + y^2 + z^2 = 1$ closest to and farthest from the point $(1, 2, 1)$.

(b) [10%] Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{z^3}{\sqrt{x^2+y^2}} \, dz \, dy \, dx$.

(4)

(a) [10%] Is the vector field $\mathbf{f}(x, y) = (4x^2 - 4y^2, \ln y - 8xy)$ conservative?

(b) [15%] A particle is moving along the parabola $y = x^2 + x - 1$ subject to the vector field given as in (a). Find the work done in moving from the point $(-1, -1)$ to the point $(4, e)$ if forces are measured in newtons and distances are measured in meters?

(c) [10%] Evaluate the surface integral $\iint_\Omega (\sin x + x^3 - 2x^5) \, d\sigma$ where Ω is the part of the surface $z = x^2$ lying over $\{(x, y) : -\pi \leq x \leq \pi, 0 \leq y \leq 1\}$.

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科目：線性代數【應數系二年級】

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1. (15分) 請解下列聯立方程組

$$\begin{cases} x_1 + 2x_2 - 2x_3 + x_4 + 3x_5 = 1 \\ 2x_1 + 5x_2 - 3x_3 - x_4 + 2x_5 = 2 \\ -3x_1 - 8x_2 + 6x_3 - x_4 - 5x_5 = 1 \\ x_1 + 2x_2 - x_3 - 3x_4 - 3x_5 = 1 \\ 5x_1 + x_2 + 7x_3 - 24x_4 + 8x_5 = 2 \end{cases}$$

2. (15分) 請寫出一個 \mathbb{R}^2 上的保長變換(orthogonal matrix),使得直線 $Y = X$ 映至 $Y = 3X$.

3. (15分) 求 $3X^2 + 2Y^2 + 3Z^2 - 2XY - 2YZ$ 在 $X^2 + Y^2 + Z^2 = 1$ 之最大與最小值. (須詳細解釋為何如此求出即為最大最小值)

4. (15分) 求解 $X_1(t), X_2(t), X_3(t)$ 滿足

$$\begin{aligned} X_1' &= X_1 - X_2 - X_3 \\ X_2' &= -X_1 + X_2 - X_3 \\ X_3' &= -X_1 - X_2 + X_3 \end{aligned}$$

where X_i' denote the derivative of X_i .

5. (20分) 設 L_1, L_2 為過原點之直線, 且與 x 軸之夾角分別為 $\frac{\theta_1}{2}$ 與 $\frac{\theta_2}{2}$, 問:

(1) 先對 L_1 鏡射再對原點旋轉 α 角之合成為旋轉或鏡射? 若旋轉, 請找出其旋轉角; 若鏡射, 請找出其鏡射軸.

(2) 先對 L_1 鏡射再對 L_2 鏡射之合成為旋轉或鏡射? 若旋轉, 請找出其旋轉角; 若鏡射, 請找出其鏡射軸.

6. (20分) We denote the determinant of X by $\det X$, and trace of X by $\text{trace} X$ for all $X \in M_n(\mathbb{R})$. Suppose that $P, Q \in M_n(\mathbb{R})$ is similar.

證明

(a) P, Q 之 characteristic polynomial 必相同.

(b) P, Q 之 eigenvalue 必相同.

(c) $\det P = \det Q$.

(d) $\text{trace} P = \text{trace} Q$.

~ 全卷完 ~