國立中山大學104學年度轉學考招生考試試題

科目名稱：微積分【應數系二年級】　　　　　　　　　　　　題號：724001

※本科目依簡章規定「不可以」使用計算機　　　　　　　　　共1頁第1頁

1. (15%) 考慮以下之函數極值問題:

$$\min\ z = x^2 + 2xy + w^2.$$

其中限制條件如下

$$2x + y + 3w = 24 \quad \text{及} \quad x + w = 8.$$

試以拉格朗日乘子法 (Lagrange multipliers) 解之.

2. (15%) 給定曲面

$$z = \cos(x + y).$$

求在點 $(x_0, y_0, z_0) = (\frac{\pi}{4}, \frac{\pi}{4}, 0)$ 處, 曲面的切平面和法線的方程式.

3. 計算二重積分

$$I = \iint_D x^2 + y^2\, d\sigma.$$

其中

(10%) (a) $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\}$ 為圓盤.

(10%) (b) $D = \{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 9\}$ 為圓環.

4. (20%) 計算迴路積分

$$I = \oint_L (x + y)\, ds.$$

其中 L 為三角形 $\triangle OAB$ 的三條有向邊 $\overline{OA}$, $\overline{AB}$ 和 $\overline{BO}$. 這裡, 三個角點分別是

$$O = (0,0),\ A = (1,0) \text{ 和 } B = (0,1).$$

5. (10%) (a) 證明 $\lim_{n\to\infty} \sqrt[n]{n!} = +\infty$.

(5%) (b) 對於冪級數 $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$, 證明其收斂半徑為 $+\infty$.

6. (10%) (a) 證明連續函數 f 會將閉區間 [a,b] 映成為閉區間 [c,d].

(5%) (b) 證明一對一的連續函數 f 會將開區間 (a,b) 映成為開區間 (c,d).

國立中山大學104學年度轉學生招生考試試題

題號：724002

科目：線性代數【應用數學系學士班二年級】

注意事項：本試卷共10題計算題，每一題10分。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. A Givens rotation is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ used in computer programs to create a zero entry in a vector (usually a column of a matrix). The standard matrix of a Givens rotation in $\mathbb{R}^2$ has the form

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}, \quad a^2 + b^2 = 1$$

Find a and b such that $\left[\begin{smallmatrix} 4 \\ 3 \end{smallmatrix}\right]$ is rotated into $\left[\begin{smallmatrix} 5 \\ 0 \end{smallmatrix}\right]$.

2. Let A_n be the $n \times n$ matrix with 0's on the main diagonal and 1's elsewhere. Find A_n^{-1}.

3. Let $A = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \cdots & a \end{bmatrix}$ be an $n \times n$ matrix. Find the determinant of A.

4. Define: $C[0,1] \to C[0,1]$ as follows: For $\mathbf{f}$ in $C[0,1]$. Let $T(\mathbf{f})$ be the antiderivative F of $\mathbf{f}$ such that $F(0) = 0$. Show that T is a linear transformation, and describe the kernel of T.

5. Diagonalize the matrix $\begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

6. The trace of a square matrix A is the sum of the diagonal entries in A and is denoted by $\operatorname{tr} A$. Show that if A and B are similar, then $\operatorname{tr} A = \operatorname{tr} B$.

7. Find the closest point to $\mathbf{y} = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}$ in the subspace W spanned by $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

8. The columns of $Q = \begin{bmatrix} 5/6 & -1/6 \\ 1/6 & 5/6 \\ -3/6 & 1/6 \\ 1/6 & 3/6 \end{bmatrix}$ were obtained by applying the Gram-Schmidt process to the columns of $A = \begin{bmatrix} 5 & 9 \\ 1 & 7 \\ -3 & -5 \\ 1 & 5 \end{bmatrix}$. Find an upper triangular matrix R such that $A = QR$.

9. Classify the quadratic form $3x_1^2 - 4x_1x_2 + 6x_2^2$. Then make a change of variable, $\mathbf{x} = P\mathbf{y}$, that transforms the quadratic form into one with no cross-product term. Write the new quadratic form.

10. Let $A(\theta) = \left[\begin{smallmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{smallmatrix}\right]$ and $B(\theta) = \left[\begin{smallmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{smallmatrix}\right]$. Compute

$$A(10°)B(20°)A(30°)B(40°)A(50°)B(60°)A(70°)B(80°)A(90°)$$