國立中山大學103學年度轉學考招生考試試題

科目名稱：微積分【應數系二年級】　　　　　　　　　　　　　　題號：724001

※本科目依簡章規定「不可以」使用計算機　　　　　　　　　　　共1頁第1頁

1.

(a) If $x^2y + y^3 = 2$ and $y(1) = 1$. Find $y'(1)$ and $y''(1)$ by implicit differentiation. [10%]

(b) Find all relative extreme values of $f(x) = 2x^3 - 3x^2 - 12x + 4$. [10%]

2.

(a) Evaluate $\int_0^{\pi/2} \cos^2 x \, \sin x \, dx$. [10%]

(b) Evaluate $\int \frac{x}{x^2+x-2} dx$. [10%]

3.

(a) Find the interval of convergence of $\sum_{n=2}^{\infty} n(n-1)(x-1)^n$. [10%]

(b) Find the sum in (a) for x inside the interval of convergence. [10%]

4.

(a) Find the shortest distance from the point $(2, 0, 1)$ to the plane $x + y + z = 1$. [10%]

(b) Find the maximum and minimum values of $f(x, y, z) = 2x + y + z + 4$ subject to the constraint $x^2 + y^2 + z^2 = 6$. [10%]

5.

(a) Evaluate the triple integral $\int\int\int_{\Omega}(x + 2z)dV$, where $\Omega = \{(x, y, z) | 0 \leq x, \ 0 \leq y, \ 1 \leq x^2 + y^2 + z^2 \leq 4\}$ [10%]

(b) Evaluate the line integral $\int_C (y\, dx - x\, dy)$, where C is the curve $x = t^2$, $y = e^{t^2}$, $0 \leq t \leq 1$, from $(0, 1)$ to $(1, e)$. [10%]

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科目名稱：線性代數【應數系二年級】 題號：742002

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注意事項：本試卷共10題計算題，每一題10分。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Find the value(s) of h for which the vectors $\begin{bmatrix}3\\-6\\1\end{bmatrix}$, $\begin{bmatrix}-6\\4\\-3\end{bmatrix}$, $\begin{bmatrix}9\\h\\3\end{bmatrix}$ are linearly dependent.

2. Find the inverses of the matrix $\begin{bmatrix}1 & 0 & -2\\-3 & 1 & 4\\2 & -3 & 4\end{bmatrix}$, if they exist.

3. Let $A = \begin{bmatrix}a & b & b & \cdots & b\\ b & a & b & \cdots & b\\ b & b & a & \cdots & b\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ b & b & b & \cdots & a\end{bmatrix}_{n\times n}$. Evaluate $\det A$.

4. Determine whether the sets of polynomials $5-3t+4t^2+2t^3$, $9+t+8t^2-6t^3$, $6-2t+5t^2$, t^3 form a basis for $\mathbb{P}_3$.

5. Let $P = \begin{bmatrix}1 & 2 & -1\\-3 & -5 & 0\\4 & 6 & 1\end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix}-2\\2\\3\end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix}-8\\5\\2\end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix}-7\\2\\6\end{bmatrix}$. Find a basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for $\mathbb{R}^3$ such that P is the change-of-coordinates matrix from $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

6. Diagonalize the matrix $\begin{bmatrix}3 & 1 & 1\\1 & 3 & 1\\1 & 1 & 3\end{bmatrix}$.

7. Let J be the $n\times n$ matrix of all 1's, and consider $A = (a-b)I + bJ$. Find the eigenvalues of A.

8. Let $\mathbf{x} = \begin{bmatrix}5\\-9\\5\end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix}-3\\-5\\1\end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix}-3\\2\\1\end{bmatrix}$. Find the distance from $\mathbf{x}$ to the plane in $\mathbb{R}^3$ spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$.

9. Find an singular value decomposition of the matrix $\begin{bmatrix}2 & 3\\0 & 2\end{bmatrix}$.

10. Determine if the set of points $\begin{bmatrix}1\\3\\1\end{bmatrix}$, $\begin{bmatrix}0\\-1\\-2\end{bmatrix}$, $\begin{bmatrix}2\\5\\2\end{bmatrix}$, $\begin{bmatrix}3\\5\\0\end{bmatrix}$ is affinely dependent. If so, construct an affine dependence relation for the points.