國立中山大學 102 學年度轉學考招生考試試題

科目名稱：微積分【應數系二年級】　　　題號：724001

※本科目依簡章規定「不可以」使用計算機　　　共2頁第1頁

說明： (1) 本考試卷共有五大題目(每一大題均有若干小題)，每一大題均為 20 分；
(2) 作答必需寫在作答紙上(不可寫在試卷紙上)。

QUESTION 1 (20 marks)

Find the following

(1.1) $\frac{dy}{dx}$, where x and y satisfy the relation $\sin(xy)+e^{2+xy}+5=0$. [05 marks]

(1.2) The gradient of the function $f(x,y,z)=\ln(1+xyz)$ at the point $(1,1,1)$ and the directional derivative of the same function at the same point in the direction $u=(-1,-1,1)$. [05 marks]

(1.3) The divergence, $\operatorname{div}(\vec{F})$, and the curl, $\operatorname{curl}(\vec{F})$, of the vector field $\vec{F}$ which is given by $\vec{F}(x,y,z)=x\sin y\vec{i}+y\sin z\vec{j}+z\sin x\vec{k}$. [05 marks]

(1.4) The equation of the tangent plane to the ellipsoid $x^2+\frac{1}{2}y^2+\frac{1}{3}z^2=1$ at the point $(\frac{1}{\sqrt{6}},1,1)$. [05 marks]

QUESTION 2 (20 marks)

(2.1) Show that the series $\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{k^3+1}}$ is absolutely convergent. [05 marks]

(2.2) Determine convergence or divergence of the series $\sum_{k=0}^{\infty}\frac{k^e}{e^k}$. [05 marks]

(2.3) Find the convergence interval of the series $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{\sqrt{k}}x^k$. [05 marks]

(2.4) Find the power series of the function $f(x)=\sin x+\frac{1}{1+x}$ around $x=0$. [05 marks]

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QUESTION 3 (20 marks)

(3.1) Show that the equation $\frac{3}{2}x+\sin x-1=0$ has exactly one real root. [05 marks]

(3.2) Use the Lagrangian multipliers method to find the minimum and maximum values of The function $f(x,y,z)=x+y+z$ on the unit sphere $x^2+y^2+z^2=1$. [15 marks]

QUESTION 4 (20 marks)

Evaluate the following integrals:

(4.1) $\int_0^1\int_{1-y}^1 \sin(x^2)dxdy$. [10 marks]

(4.2) $\iiint\limits_T xyzdxdydz$, where T is the solid which is above the plane $z=0$, below the plane $y=z$, and inside the cylinder $x^2+y^2=4$. [10 marks]

QUESTION 5 (20 marks)

Find the following:

(5.1) The area of the part of the sphere $x^2+y^2+z^2=1$ that is cut out by the cone $z=\sqrt{x^2+y^2}$ in the upper half-plane $z\geq 0$. [10 marks]

(5.2) Use Green's Theorem to evaluate the line integral $\oint_C (xy-\frac{1}{2}y^2)dx+(\frac{1}{2}x^2+xy)dy$, where C is the boundary of the region in the first quadrant enclosed by the ellipse $x^2+4y^2=1$, the line $y=\frac{1}{2}x$ and the x-axis. [10 marks]

--End of the Question Paper--

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國立中山大學 102 學年度轉學考招生考試試題

科目名稱：線性代數【應數系二年級】 題號：724002

※本科目依簡章規定「不可以」使用計算機 共1頁第1頁

請標示題號並詳細陳述計算或證明.

1. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal. (10%)

2. Define $T : \mathbf{R}^4 \to \mathbf{R}^3$ by $T(x, y, z, w) = (x + y, x - y + z, x - 2y - w)$. Find bases of the kernel of T and the image space of T. (15%)

3. Let $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$, $k < n$, be an independent subset of $\mathbf{R}^n$, and A be an $n \times n$ matrix. Suppose that $\{A\mathbf{u}_1, A\mathbf{u}_2, \ldots, A\mathbf{u}_k\}$ is independent. Is A necessarily invertible? (10%)

4. Let T be a linear operator on $\mathbf{R}^2$. If the matrix representation of T relative to the ordered basis $B = \{(1, 1), (-1, 0)\}$ is $\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$.
   (a) Find $T(-1, 1)$. (10%)
   (b) Let $D = \{(-1, 1), (0, 1)\}$ be another ordered basis of $\mathbf{R}^2$. Find the matrix representation of T relative to D. (10%)

5. Let $< \mathbf{u}, \mathbf{v} >= x_1y_1 + 2x_1y_2 - x_2y_1 + x_2y_2$, where $\mathbf{u} = (x_1, x_2)^T$ and $\mathbf{v} = (y_1, y_2)^T$.
   (a) Find a matrix A such that $< \mathbf{u}, \mathbf{v} >= (A\mathbf{u})^T\mathbf{v}$. (10%)
   (b) Does $< \cdot, \cdot >$ define an inner product on $\mathbf{R}^2$? (10%)

6. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.
   (a) Find the minimal polynomial of A. (5%)
   (b) Find a Jordan form for A. (10%)
   (c) Find a matrix P such that $P^{-1}AP$ is the Jordan form in (b). (10%)