國立中山大學101學年度轉學生招生考試試題

科目：微積分【應數系學士班二年級】

題號：7022

共 1 頁 第 1 頁

注意事項：本試卷共20題填充題，每一題5分。

1. Find $dy/dx =$ ___①___ given that $y^3 + y^2 - 5y - x^2 = -4$.

2. Find the extrema = ___②___ of $f(x) = x^3 - \frac{3}{2}x^2$ on the interval $[-1, 2]$.

3. Find two positive numbers = ___③___ which satisfy that the second number is the reciprocal of the first number and the sum is a minimum.

4. Evaluate $\int_1^4 (3 - |x - 3|)\, dx =$ ___④___.

5. Evaluate $\int \sec(1-x)\tan(1-x)\, dx =$ ___⑤___.

6. Find $(f^{-1})'(a) =$ ___⑥___ where $f(x) = x^3 - 1$ and $a = 26$.

7. Find the volume = ___⑦___ of the solid generated by revolving the region bounded by the graphs of $y = \frac{1}{\sqrt{x+1}}$, $y = 0$, $x = 0$, and $x = 4$ about the x-axis.

8. Find the area = ___⑧___ of the surface generated by revolving $y = \frac{x}{2}$, $0 \le x \le 6$ about the x-axis.

9. Evaluate $\int \frac{x+2}{x^2 - 4x}\, dx =$ ___⑨___.

10. Evaluate $\lim_{x \to 1^+} \frac{\int_1^x \cos\theta\, d\theta}{x - 1} =$ ___⑩___.

11. Determine the convergence or divergence = ___⑪___ of the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

12. Find the interval = ___⑫___ of convergence of the power series $\sum_{n=0}^{\infty} \left(\frac{x}{k}\right)^n$, $k > 0$.

13. Find the Maclaurin series = ___⑬___ for the function $f(x) = \frac{1}{(1+x)^2}$.

14. Find the arc length = ___⑭___ of the curve $x = t^2$ and $y = 2t$, $0 \le t \le 2$.

15. Find the area = ___⑮___ of the common interior of $r = 4\sin 2\theta$ and $r = 2$.

16. Determine whether the statement is true or false = ___⑯___. Every two lines in space are either intersecting or parallel.

17. Find the directional derivative = ___⑰___ of $f(x, y) = x^2 + 3y^2$ at $P(1, 1)$ in the direction of $Q(4, 5)$.

18. Find the minimum value = ___⑱___ of $f(x, y) = 3x + y + 10$ subject to the constraint $x^2 y = 6$.

19. Evaluate $\int_0^4 \int_0^{\sqrt{4y - y^2}} x^2\, dx\, dy =$ ___⑲___.

20. Find the volume = ___⑳___ of the solid inside both $x^2 + y^2 + z^2 = a^2$ and $(x - a/2)^2 + y^2 = (a/2)^2$.

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科目：線性代數 【應數系學士班二年級】

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共 01 頁 第 01 頁

Answer each of the following problems. Show details of your work.

1. (a) Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 2 & -1 & -1 \end{bmatrix}$. Find a basis for the null space of A. (8%)

   (b) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Is A invertible? Find A^{-1} if it exists. (12%)

2. (a) Is it possible to find a 2×2 matrix A such that $A^2 + I = 0$? How about for 3×3 matrix? (5%)

   (b) Let A be an orthogonal matrix.
   (i) Show that $\det A = 1$ or $\det A = -1$. (3%)
   (ii) If $\det A = -1$, show that $I + A$ is not invertible. (7%)

3. Let $A = \begin{bmatrix} 4 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & -2 \end{bmatrix}$. Find an orthogonal P such that $P^{-1}AP = D$ is diagonal. Also, find D. (15%)

4. Let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation. Prove or disprove: If $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ is linear independent, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is also independent. (10%)

5. (a) Define $T : \mathbf{R}^2 \to \mathbf{R}^2$ by $t(x, y) = (x + 2y, x - y)$. Find the matrix of T relative to the ordered basis $B = \{(1, 1), (2, -1)\}$. (8%)

   (b) A linear operator $T : \mathbf{R}^3 \to \mathbf{R}^3$ sends $(1, 0, 0)$ to $(2, 0, -1)$, $(1, 1, 0)$ to $(1, 1, 0)$, and $(0, 0, 1)$ to $(3, 2, 1)$. Write its matrix relative to the standard basis. (12%)

6. If A is an $n \times n$ matrix such that $A^2 = A$. Let $\mathbf{U} = \{\mathbf{v} | A\mathbf{v} = \mathbf{v}\}$ and $\mathbf{W} = \ker A$. Prove or disprove: $\mathbf{R}^n = \mathbf{U} \oplus \mathbf{W}$. (20%)

End of Paper