國立中山大學 100 學年度學士班轉學生招生考試試題

科目：微積分【應用數學系二年級】

共十題，每題 10 分。答題時，每題都必須寫下題號與詳細步驟。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Find the limit: $\lim_{x\to 0} \dfrac{|x|}{x}$.

2. The edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

3. Prove that if f is differentiable on $(-\infty, \infty)$ and $f'(x) < 1$ for all real numbers, then f has at most one fixed point. A fixed point of a function f is a real number c such that $f(c) = c$.

4. Show that the function $f(x) = \int_0^{1/x} \frac{1}{t^2+1}\,dt + \int_0^{x} \frac{1}{t^2+1}\,dx$ is constant for $x > 0$.

5. Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ in the first and second quadrants.

6. Sketch the graph of $g(x) = \begin{cases} e^{-1/x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ and determine $g'(0)$.

7. Find the radius and interval of convergence of
$$\sum_{n=1}^{\infty} \left[\frac{2\cdot 4\cdot 6\cdots 2n}{3\cdot 5\cdot 7\cdots (2n+1)}\right] x^{2n+1}.$$

8. Find the Maclaurin series for the function $g(x) = \dfrac{x}{1-x-x^2}$.

9. Show that any tangent plane to the cone
$$z^2 = a^2x^2 + b^2y^2$$
passes through the origin.

10. Find the volume of the region of points (x, y, z, w) such that
$$x^2 + y^2 + z^2 + w^2 \leq a^2.$$

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National Sun Yat-sen University
Department of Applied Mathematics
Examination: Linear Algebra
Question Paper

Date: July 7, 2011
Mark: 100
Time: 80 minutes
Note: This question paper is composed of five (5) questions. Attempt all of them.

Question One

[20 marks]

(1.1) Prove the Cauchy-Schwartz inequality in the Euclidean n-space $\mathbb{R}^n$:

$$|\langle x, y\rangle| \le \|x\|\|y\|, \quad x, y \in \mathbb{R}^n, \qquad (*)$$

where $\langle\cdot,\cdot\rangle$ is the (standard) dot product on $\mathbb{R}^n$ and $\|\cdot\|$ is the norm induced by the dot product $\langle\cdot,\cdot\rangle$. [15 marks]

(1.2) Further show that the equality $=$ in (*) holds if and only if there is a real number $t \in \mathbb{R}$ such that either $y = tx$ or $x = ty$. [5 marks]

Question Two

[20 marks]

Determine whether the matrix A given below is diagonalizable. If it is so, find a matrix U that diagonalizes A; that is, $U^{-1}AU = \Lambda$ is a diagonal matrix.

$$A = \begin{bmatrix} -3 & 2 & 3 \\ -1 & 1 & 1 \\ -4 & 1 & 4 \end{bmatrix}.$$

Question Three

[20 marks]

Find the value(s) of a so that the vector $x = \begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}$ is an eigenvector of A^{-1}, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

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Question Four [20 marks]

(4.1) Find the values of a and b so that the matrices

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 2 & a & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & b \end{bmatrix}$$

are similar. [10 marks]

(4.2) Prove that if a square matrix A satisfies the equation $2A^2 - 3A - 5I = 0$, then none of the eigenvalues of the matrix $2A + I$ are zero. [10 marks]

Question Five [20 marks]

(5.1) Assume that A is an $m \times n$ real matrix. Prove that the matrices $A^T A$ and AA^T have the same positive eigenvalues. [Here A^T is the transpose of A.] [10 marks]

(5.2) Find the singular value decomposition (SVD) of the matrix A, where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}.$$

[10 marks]

- End of Question Paper -