

## 國立中山大學九十三年度轉學生招生考試試題

科目：微積分【物理系二年級、電機系二年級、材光系二年級、海工系二年級】 共 | 頁第 | 頁

Full marks are 100; the marks are indicated within questions.

I.

(a) [10%] Evaluate  $\int_0^{\infty} e^{-st} \cos at \, dt$  for  $a, s > 0$ .

(b) [15%] Evaluate  $\int_0^1 x^3 \sqrt{1-x^2} \, dx$ .

II.

(a) [10%] Find the Taylor series for  $e^x$  at  $x = -1$ .

(b) [15%] Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^{n+1}}$ .

III.

(a) [10%] Let  $f(x, y) = x^3y$ . Calculate the directional derivative of  $f$  in the direction of the vector  $\mathbf{v} = \langle 1, -1 \rangle$  at the point  $(1, 2)$ .

(b) [15%] Find the mass of the ball  $x^2 + y^2 + z^2 \leq 1$  if its density at a point is the square root of the distance from the point to the origin.

IV.

(a) [10%] Evaluate the line integral  $\int_C \mathbf{f} \cdot d\mathbf{r}$  where  $\mathbf{f}(x, y) = (x^2 + xy^2, x^2y + y^2)$  and  $C$  is the parabola  $y = x^2 - 2x$  from  $(0, 0)$  to  $(2, 0)$ .

(b) [15%] Evaluate the surface integral  $\int_S x \, d\sigma$  where  $S = \{(x, y, z) : z = x^2, 0 \leq x \leq 1, 0 \leq y \leq 2\}$ .

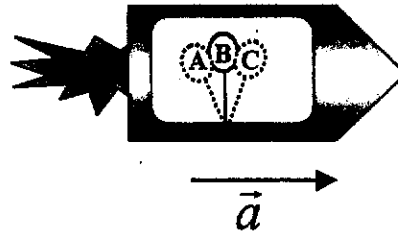
# 國立中山大學九十三年學年度轉學生招生考試試題

科目：普通物理【物理系二年級、電機系二年級、材光系二年級】

共二頁第 / 頁

## Selections (5% for each question)

1. A tank is filled up with pure water and is carried by a rocket acceleration horizontally to the right as shown in the figure 1. A balloon is fixed by a wire attached to the bottom of the water tank, where will the balloon stabilize at (1) A, (2) B, (3) C and (4) the bottom of the tank.



2. An ambulance running with a velocity of  $\bar{v}_0$  away from an observer who stands still on the ground. If the emergency alarm is of frequency  $f$ , what frequency ( $f'$ ) should the observer hear? (a)  $f' = f\left(1 - \frac{v_0}{v}\right)$ , (b)  $f' = f\left(1 + \frac{v_0}{v}\right)$ , (c)

$$f' = f\left(\frac{1}{1 - \frac{v_0}{v}}\right), \text{ (d) } f' = f\left(\frac{1}{1 + \frac{v_0}{v}}\right).$$

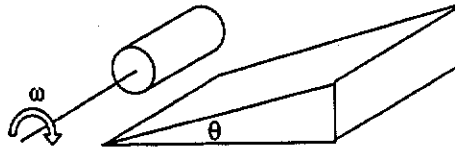
3. What will happen when a balloon filled with He gas is warmed up? (a) He atoms moves slower, (b) He atoms moves faster, (c) He atoms moves toward the same direction, (d) the speed of He atoms does not change.
4. A conductor is charged with  $Q$ . What is the electric field ( $E$ ) and the electric potential ( $V$ ) inside the conductor? (a)  $E = \text{constant}$ ,  $V = \text{constant}$ ; (b)  $E = \text{constant}$ ,  $V = 0$ ; (c)  $E = 0$ ,  $V = 0$ ; (d)  $E = 0$ ,  $V = \text{constant}$ .
5. Two parallel wires carry the same current that flow in the same direction, the two wires will (a) remain still; (b) reject each other; (c) attract with each other; (d) become twisted.
6. Total reflection will occurred at an interface when light is traveling from (a) the medium with small  $n$  to the medium with large  $n$ , (b) the medium with large  $n$  to the medium with small  $n$ , (c) mediums with the same refractive index, (d) the air into water. ( $n$  is the refractive index)

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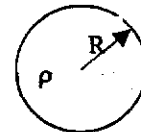
科目：普通物理【物理系二年級、電機系二年級、材光系二年級】 共二頁 第 2 頁

## Calculations (total 70%)

1. (15%) A rod of mass  $m$ , radius  $r$  and length of  $L$  rotates along its axis with a constant angular velocity  $\omega$ . If the friction coefficient between the rod and the inclined plane is  $\mu$ , how far can the rod climb when the rod is placed on the inclined plane slightly.



2. (10%) A high pressure tank is filled of 50 atm of He gas at 300K, What will the gas temperature be when the valve of the tank is opened suddenly and the gas experiences adiabatic expansion to its 1 atm environment?
3. (25%) A sphere with a radius  $R$  contains charges distributed as  $\rho = br$  where  $\rho$  and  $r$  are the charge density and the distance to the center of the sphere. Please calculate the electric field ( $\vec{E}$ ) and the electric potential ( $V$ ) at  $r < R$ ,  $r = R$  and  $r > R$ .



4. (20%) A large parallel-plate capacitor with uniform surface charge  $\sigma$  on the upper plate and  $-\sigma$  on the lower is moving with a constant speed  $v$ , as shown in the right figure. (a) Find the magnetic field between, above and below these plates. (b) Find the magnetic and electrical force per unit area on the upper plate, including their directions. (c) At what speed  $v$  would the magnetic force balance the electrical force?

