注意事項：本試卷共20題填充題，每一題5分。

1. Determine the slope = ____①____ of the graph of $3(x^2 + y^2)^2 = 100xy$ at the point $(3, 1)$.

2. Find the extrema = ____②____ of $y = 3x^{2/3} - 2x$ on the interval $[-1, 1]$.

3. Find two positive numbers = ____③____ which satisfy that the sum of the first number squared and the second number is 54 and the product is a maximum.

4. Evaluate $\int_0^4 |x^2 - 4x + 3|\, dx = $ ____④____ .

5. Evaluate $\int \sqrt{\tan x}\, \sec^2 x\, dx = $ ____⑤____ .

6. Find $(f^{-1})'(a) = $ ____⑥____ where $f(x) = x^3 + 2x - 1$ and $a = 2$.

7. Find the volume = ____⑦____ of the solid generated by revolving the region bounded by the graphs of $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, and $x = 3$ about the x-axis.

8. Find the area = ____⑧____ of the surface generated by revolving $y = \sqrt{9 - x^2}$, $-2 \leq x \leq 2$ about the x-axis.

9. Evaluate $\int \dfrac{3x - 4}{(x - 1)^2}\, dx = $ ____⑨____ .

10. Evaluate $\lim_{x \to 0} \dfrac{x}{\arctan 2x} = $ ____⑩____ .

11. Determine the convergence or divergence = ____⑪____ of the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^2}$.

12. Find the interval = ____⑫____ of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{k(k+1)(k+2)\cdots(k+n-1)x^n}{n!}$, $k \geq 1$.

13. Find the Maclaurin series = ____⑬____ for the function $f(x) = \dfrac{1}{\sqrt{1-x}}$.

14. Find the arc length = ____⑭____ of the curve $x = t^2 + 1$ and $y = 4t^3 + 3$, $-1 \leq t \leq 0$.

15. Find the area = ____⑮____ of the common interior of $r = 3 - 2\sin\theta$ and $r = -3 + 2\sin\theta$.

16. Determine whether the statement is true or false = ____⑯____ . If two lines L_1 and L_2 are parallel to a plane P, then L_1 and L are parallel.

17. Find the directional derivative = ____⑰____ of $g(x, y, z) = xye^z$ at $P(2, 4, 0)$ in the direction of $Q(0, 0, 0)$.

18. Find the minimum value = ____⑱____ of $f(x, y) = \sqrt{x^2 + y^2}$ subject to the constraint $2x + 4y - 15 = 0$.

19. Evaluate $\int_0^2 \int_0^{\sqrt{4 - x^2}} \sin\sqrt{x^2 + y^2}\, dy\, dx = $ ____⑲____ .

20. Find the volume = ____⑳____ of the solid bounded above $z = 2x$ and below by $z = 2x^2 + 2y^2$.

選擇題 60%（單選，每題5分，答錯不倒扣）

1. A car travels 40 kilometers at an average speed of 80 km/h and then travels 40 kilometers at an average speed of 40 km/h. The average speed of the car for this 80 km trip is: (a) 40 km/h (b) 45 km/h (c) 48 km/h (d) 53 km/h (e) 80 km/h.

2. A racing car traveling with constant acceleration increases its speed from 10 m/s to 30 m/s over a distance of 80 m. How long does this take? (a) 2.0 s (b) 4.0 s (c) 5.0 s (d) 8.0 s (e) The time cannot be calculated since the speed is not constant.

3. A ball is thrown horizontally from the top of a 20-m high hill. It strikes the ground at an angle of 45°. With what speed was it thrown? (The gravitational acceleration constant g=9.8 m/s^2) (a) 14.2 m/s (b) 19.8 m/s (c) 28.6 m/s (d) 32.4 m/s (e) 40 m/s.

4. A 90-kg man stands in an elevator that has a downward acceleration of 1.4 m/s². The force exerted by him on the floor is about: (The gravitational acceleration constant g=9.8 m/s^2) (a) zero (b) 90 N (c) 756 N (d) 882 N (e) 1008 N.

5. A 12-kg crate rests on a horizontal surface and a boy pulls on it with a force that is 30° below the horizontal. If the coefficient of static friction is 0.40, the minimum magnitude force he needs to start the crate moving is: (a) 44 N (b) 47 N (c) 54 N (d) 56 N (e) 71 N.

6. A 0.50-kg block attached to an ideal spring with a spring constant of 80 N/m oscillates on a horizontal frictionless surface. The total mechanical energy is 0.12 J. The greatest extension of the spring from its equilibrium length is: (a) 1.5×10^{-3} m (b) 3.0×10^{-3} m (c) 0.039 m (d) 0.054 m (e) 18 m.

7. A 0.20-kg particle moves along the x axis under the influence of a stationary object. The potential energy is given by $U(x) = (8.0 J/m^2)x^2 + (2.0 J/m^4)x^4$, where x is in coordinate of the particle. If the particle has a speed of 5.0 m/s when it is at $x = 1.0$ m, its speed when it is at the origin is: (a) 0 (b) 2.5 m/s (c) 5.7 m/s (d) 7.9 m/s (e) 11 m/s.

8. The pull P is just sufficient to keep the 14-N block and the weightless pulleys in equilibrium as shown. The tension T in the upper cable is: (a) 14 N (b) 28 N (c) 16 N (d) 9.33 N (e) 18.7 N.
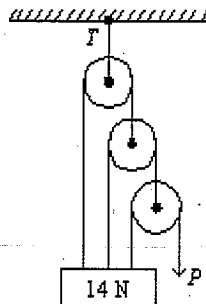

9. A Carnot engine operates between 200°C and 20°C. Its maximum possible efficiency is: (a) 90% (b) 100% (c) 38% (d) 72% (e) 24%.

10. A plane parallel plate capacitor has plates of 10 cm^2 area that are 1.0 mm apart. At an instant when charge is being accumulated on the plates at a rate of 12 nC/s, the displacement current between the plates is (a) 1.06×10^{-16} A (b) 1.2×10^{-8} A (c) 8.85×10^{-9} A

(d)1.00 A (e)1.36 A.

11. If a = 1.0 cm, b = 3.0 cm, and I = 30 A, what is the magnitude of the magnetic field at point P? (Permeability of free space $\mu_0=4\pi\times10^{-7}$ T·m/A) (a) 0.62 mT (b)0.59 mT (c)0.35 mT (d)0.31 mT (e)0.10 mT.
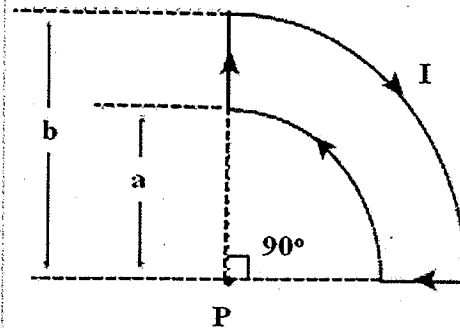
12. The phase difference between the two waves which give rise to a dark spot in a Young's double-slit experiment is (where m = integer): (a) zero (b) $2\pi m + \pi/8$ (c) $2\pi m + \pi/4$ (d) $2\pi m + \pi/2$ (e) $2\pi m + \pi$.

非選擇題 （40%）

1. A solid cube of wood of side 2a and mass M is resting on a horizontal surface. The cube is constrained to rotate about a fixed axis AB (Fig. P1). A bullet of mass m and speed v is shot at the face opposite ABCD at a height of 4a/3. The bullet becomes embedded in the cube. (a) Please show that the rotational momentum inertia of cube along one of its edge is $8Ma^2/3$. (10 分) (b) Find the minimum value of v required to tip the cube so that it falls on face ABCD. (10 分) Assume m<< M.
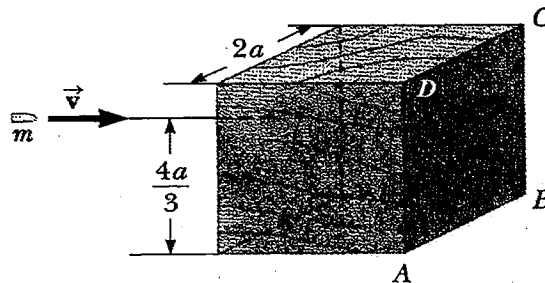

Figure P1

2. A conducting bar of length ℓ moves to the right on two frictionless rails as shown in Fig. P2. A uniform magnetic field directed into the page has a magnitude of 0.3 T. Assume R=9.00Ω and ℓ = 0.35m (a) At what constant speed should the bar move to produce an 8.50 mA current in the resistor? (7 分) (b) What is the direction of the induced current? (6 分) (c) At what rate is energy delivered to the resistor? (7 分)
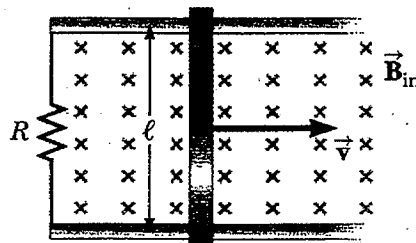

Figure P2

背面有題