國立中山大學104學年度轉學生招生考試試題

題號：735001

科目：微積分【光電系學士班二年級】

注意事項：本試卷共10題計算題，每一題10分。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Determine all values of the constant a such that the following function is continuous for all real numbers.
$$f(x) = \begin{cases} \frac{ax}{\tan x}, & x \geq 0 \\ a^2 - 2, & x < 0 \end{cases}$$

2. Find the length of the longest pipe that can be carried level around a right-angle comer at the intersection of two corridors of widths 4 feet and 6 feet.

3. Evaluate the limit
$$\lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{n}}{n^{3/2}}.$$

4. Find the area under the curve
$$y = \frac{1}{\sin^2 x + 4\cos^2 x}$$
between $x = 0$ and $x = \pi/4$.

5. The graph of $y = f(x)$ passes through the origin. The arc length of the curve from $(0,0)$ to $(x, f(x))$ is given by
$$s(x) = \int_0^x \sqrt{1 + e^t}\, \mathrm{d}t.$$
Identify the function f.

6. Let n be a positive integer. Evaluate the integral
$$\int_0^{\pi/2} \frac{\sin^n x}{\cos^n x + \sin^n x}\, \mathrm{d}x.$$

7. Evaluate $\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{2n+1}(2n+1)!}$.

8. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

9. Find the directional derivative of $f(x, y) = x^2 y$ at $(-5, 5)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

10. Evaluate the integral $\int_0^{\infty} \int_0^{\infty} \frac{1}{(1+x^2+y^2)^2}\, \mathrm{d}x\, \mathrm{d}y$.

國立中山大學104學年度轉學考招生考試試題

科目名稱：普通物理【光電系二年級】　　題號：735002

※本科目依簡章規定「不可以」使用計算機　　

單選題, 共20題, 每題5分, 總分100分, 不作答0分, 答錯倒扣1分.

1. A sledge (including load) weighs 5000 N. It is pulled on level snow by a dog team exerting a horizontal force on it. The coefficient of kinetic friction between sledge and snow is 0.05. How much work is done by the dog team pulling the sledge 1000 m at constant speed?
(A) 0 J (B) 2.5×10^5 J (C) 5.0×10^5 J (D) 2.5×10^6 J (E) 5.0×10^6 J

2. A man pushes an 80-N crate a distance of 5.0 m upward along a frictionless slope that makes an angle of 30° with the horizontal. His force is parallel to the slope. If the speed of the crate decreases at a rate of 1.5 m/s^2, then the work done by the man is:
(A) 300 J (B) 61 J (C) 140 J (D) 200 J (E) 400 J

3. A small object of mass m, on the end of a light cord, is held horizontally at a distance r from a fixed support as shown. The object is then released. What is the tension in the cord when the object is at the lowest point of its swing?

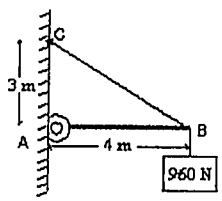
(A) $mg/2$ (B) mg (C) $2\,mg$ (D) $3\,mg$ (E) mgr

4. A rocket exhausts fuel with a velocity of 1500 m/s, relative to the rocket. It starts from rest in outer space with fuel comprising 80 per cent of the total mass. When all the fuel has been exhausted its speed is: (ln 10=2.303, ln 3=1.098, ln 2=0.693)
(A) 3600 m/s (B) 2400 m/s (C) 1200 m/s (D) 880 m/s (E) 400 m/s

5. A disk with a rotational inertia of 2.0 kg·m^2 and a radius of 0.40 m rotates on a frictionless fixed axis perpendicular to the disk faces and through its center. A force of 5.0 N is applied tangentially to the rim. The angular acceleration of the disk is:
(A) 0.40 rad/s^2 (B) 0.60 rad/s^2 (C) 1.0 rad/s^2 (D) 2.5 rad/s^2 (E) 10 rad/s^2

6. A uniform disk, a thin hoop, and a uniform sphere, all with the same mass and same outer radius, are each free to rotate about a fixed axis through its center. Assume the hoop is connected to the rotation axis by light spokes. With the objects starting from rest, identical forces are simultaneously applied to the rims, as shown. Rank the objects according to their angular momenta after a given time t, least to greatest.
(A) all tie (B) disk, hoop, sphere (C) sphere, disk, hoop (D) hoop, sphere, disk (E) hoop, disk, sphere

7. A 960-N block is suspended as shown. The beam AB is weightless and is hinged to the wall at A. The tension force of the cable BC has magnitude:
(A) 720 N (B) 1200 N (C) 1280 N (D) 1600 N (E) 1400 N



8. For a planet in orbit around a star the perihelion distance is r_p and its speed at perihelion is v_p. The aphelion distance is r_a and its speed at aphelion is v_a. Which of following is true?
(A) $v_a = v_p$ (B) $v_a/r_a = v_p/r_p$ (C) $v_a r_a = v_p r_p$ (D) $v_a/r_a^2 = v_p/r_p^2$ (E) $v_a r_a^2 = v_p r_p^2$

9. Positive charge Q is distributed uniformly throughout an insulating sphere of radius R, centered at the origin. A particle with a positive charge Q is placed at $x = 2R$ on the x axis. The magnitude of the electric field at $x = R/2$ on the x axis is:
(A) $Q/72\pi\varepsilon_0 R^2$ (B) $Q/8\pi\varepsilon_0 R^2$ (C) $7Q/18\pi\varepsilon_0 R^2$ (D) $11Q/18\pi\varepsilon_0 R^2$ (E) $Q/18\pi\varepsilon_0 R^2$

10. When a 100-Hz oscillator is used to generate a sinusoidal wave on a certain string the wavelength is 10 cm. When the tension in the string is doubled the generator produces a wave with a frequency and wavelength of:

背面有題

(A) 200 Hz and 20 cm (B) 141 Hz and 10 cm (C) 100 Hz and 20 cm (D) 100 Hz and 14 cm (E) 50 Hz and 14 cm

11. A cylindrical copper rod has resistance R. It is reformed to twice its original length with no change of volume. Its new resistance is:
(A) $R/2$ (B) R (C) $2R$ (D) $4R$ (E) $8R$

12. A wire carrying a charge density of λ C/m is bent into a circle of radius r. What is the electric potential at the center of the circle?
(A) $\lambda/4\pi\varepsilon_0 r$ (B) $\lambda/4\pi\varepsilon_0$ (C) $\lambda/4\varepsilon_0$ (D) $\lambda/2\varepsilon_0$ (E) λ/ε_0

13. The maximum theoretical efficiency of a Carnot engine operating between reservoirs at the steam point and at room temperature is about:
(A) 10% (B) 20% (C) 50% (D) 80% (E) 99%

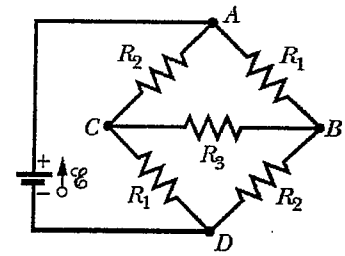
14. A particle with charge Q is on the y axis a distance a from the origin and a particle with charge q is on the x axis a distance d from the origin. The value of d for which the x component of the force on the second particle is the greatest is:
(A) 0 (B) a (C) $\sqrt{2}a$ (D) $a/2$ (E) $a/\sqrt{2}$

15. The magnetic field at any point in the xy plane is given by $\vec{B} = A\vec{r} \times \hat{k}$, where $\vec{r}$ is the position vector of the point, A is a constant, and $\hat{k}$ is a unit vector in the $+z$ direction. The net current through a circle of radius R, in the xy plane and centered at the origin is given by:
(A) $\pi AR^2/\mu_0$ (B) $2\pi AR/\mu_0$ (C) $4\pi AR^3/3\mu_0$ (D) $2\pi AR^2/\mu_0$ (E) $\pi AR^2/2\mu_0$

16. In a Young's double-slit experiment, the slit separation is doubled. This results in:
(A) an increase in fringe intensity (B) a decrease in fringe intensity (C) a halving of the wavelength (D) a halving of the fringe spacing (E) a doubling of the fringe spacing

17. In a reference frame S', the space ship is moving with velocity 0.5 c. The frame S' is moving with velocity 0.6 c relative to another frame S. What is the velocity of the spaceship with respect to frame S?
(A) 0.85 c (B)1.1 c (C) 0.99 c (D) 0.1 c (E) 0.75 c

18. In the right figure, the battery has emf $\mathscr{E}$=12.0 V, R_1 = 2000 Ω, R_2 = 3000 Ω, and R_3 = 4000 Ω. What are the potential differences between V_A and V_B.
(A) 6.20 V (B) 5.25 V (C) 2.5 V (D) 4.35 V (E) 3.75 V



19. A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire, in the unit of mm?
(A) 5.3 (B) 5.6 (C) 6.2 (D) 4.5 (E) 4.9

20. Four identical particles of mass 0.50 kg each are placed at the vertices of a 2.0 m x 2.0 m square and held there by four massless rods, which form the sides of the square. What is the rotational inertia of this rigid body about an axis that passes through the midpoint of one of the sides and is perpendicular to the plane of the square?
(A) 3.0 kg·m^2 (B) 5.0 kg·m^2 (C) 6.0 kg·m^2 (D) 7.0 kg·m^2 (E) 2.0 kg·m^2

背面有題