

# 國立中山大學 103 學年度轉學考招生考試試題

科目名稱：微積分【光電系二年級】

題號：735001

※本科目依簡章規定「不可以」使用計算機

共 1 頁第 1 頁

注意事項：本試卷共 20 題填充題，每一題 5 分。

請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Find  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x^2} = \underline{\textcircled{1}}$ .
2. Find  $\frac{d^2y}{dx^2} = \underline{\textcircled{2}}$  in terms of  $x$  and  $y$  if  $x^2y^2 - 2x = 3$ .
3. Find all asymptotes =  $\underline{\textcircled{3}}$  of  $f(x) = \frac{3x}{\sqrt{4x^2+1}}$ .
4. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 14 cubic centimeters. Find the radius =  $\underline{\textcircled{4}}$  of the cylinder that produces the minimum surface area.
5. Evaluate  $\int_0^4 |x^2 - 4x + 3| dx = \underline{\textcircled{5}}$ .
6. Evaluate  $\int \frac{-x}{(x+1)\sqrt{x+1}} dx = \underline{\textcircled{6}}$ .
7. Find the equation =  $\underline{\textcircled{7}}$  of the tangent line to  $y = e^{-2x+x^2}$  at  $(2, 1)$ .
8. Find  $\frac{d}{dx} [\sinh^{-1} x] = \underline{\textcircled{8}}$ .
9. Find the volume =  $\underline{\textcircled{9}}$  of the solid generated by revolving the plane region bounded by the graphs of  $y = x^2$  and  $y = 4x - x^2$  about the line  $x = 4$ .
10. Find the volume =  $\underline{\textcircled{10}}$  of the solid of revolution formed by revolving the circle  $x^2 + (y - 3)^2 = 4$  about the  $x$ -axis.
11. Find  $\int \cos \sqrt{x} dx = \underline{\textcircled{11}}$ .
12. Evaluate  $\lim_{x \rightarrow \infty} x^{\ln 2 / (1 + \ln x)} = \underline{\textcircled{12}}$ .
13. Determine the convergence or divergence of  $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}} = \underline{\textcircled{13}}$ .
14. Find the Maclaurin series =  $\underline{\textcircled{14}}$  for the function  $f(x) = \cos^2 x$ .
15. Find the arc length =  $\underline{\textcircled{15}}$  of the curve  $x = a(\theta - \sin \theta)$  and  $y = a(1 - \cos \theta)$  on the interval  $[0, 2\pi]$ .
16. Find the area =  $\underline{\textcircled{16}}$  of the triangle with the given vertices  $A(2, -3, 4)$ ,  $B(0, 1, 2)$ , and  $C(-1, 2, 0)$ .
17. Find the maximum value =  $\underline{\textcircled{17}}$  of the directional derivative at  $(3, 2)$ .
18. Find the minimum value =  $\underline{\textcircled{18}}$  of  $f(x, y, z) = x^2 + y^2 + z^2$  subject to constraints  $x + 2z = 6$ ,  $x + y = 12$ ,  $x \geq 0$ ,  $y \geq 0$ , and  $z \geq 0$ .
19. Evaluate  $\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2 + y^2} dx dy = \underline{\textcircled{19}}$ .
20. Find the volume =  $\underline{\textcircled{20}}$  of the solid is inside the sphere  $x^2 + y^2 + z^2 = 4$  and above the upper nappe of the cone  $z^2 = x^2 + y^2$ .

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科目名稱：普通物理【光電系二年級】

題號：723002

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共 2 頁第 1 頁

單選題，共 20 題，每題 5 分，總分 100 分，不作答 0 分，答錯倒扣 1 分。

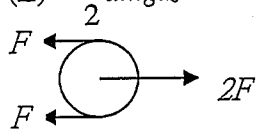
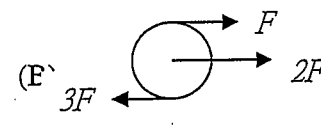
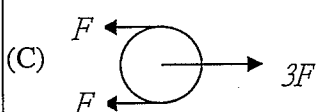
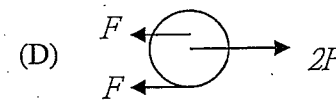
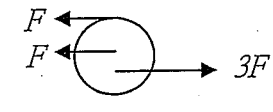
1. Four particles, labeled as  $a, b, c, d$ , move at an identical acceleration  $a\hat{y}$  and  $a > 0$ . At  $t = 0$ , the velocities are  $\vec{v}_a(0) = 12\hat{x} + 10\hat{y}$ ,  $\vec{v}_b(0) = 12\hat{x} + 8\hat{y}$ ,  $\vec{v}_c(0) = 8\hat{x} + 12\hat{y}$ , and  $\vec{v}_d(0) = 12\hat{x} + 12\hat{y}$ . Compare their magnitudes of respective displacements for time  $t > 0$ ,  $R_a, R_b, R_c, R_d$ : (A)  $R_a > R_b > R_c > R_d$ , (B)  $R_c > R_b > R_d > R_a$ , (C)  $R_d > R_a > R_b > R_c$ , (D)  $R_d > R_a > R_c > R_b$ , (E)  $R_a > R_c > R_b > R_d$ . (To avoid confusion, consider all relevant dimensions in SI units.)

2. For the position vector  $\vec{r} = (-t^2 + 2)\hat{i} + (-t^2 - t + 1)\hat{j} + (t^3 - 3t^2)\hat{k}$ , what is the magnitude of acceleration at  $t = 1$ ? (A)  $\sqrt{5}$ , (B)  $\sqrt{14}$ , (C)  $2\sqrt{2}$ , (D) 2, (E) 0. (Dimensions all in SI units.)

3. For a solid sphere of radius  $R$  and mass  $M$  rotating about an axis passing through the spherical center, what is the magnitude of rotational inertia? (A)  $\frac{1}{2}MR^2$ , (B)  $\frac{2}{3}MR^2$ , (C)  $\frac{1}{4}MR^2$ , (D)  $\frac{2}{5}MR^2$ , (E)  $MR^2$ .

4. A man of mass  $m$ , initially at rest, falls from a point at  $(d, 0, 0)$ . If the gravitational acceleration is  $\vec{a}_g = -g\hat{z}$ , find the angular momentum of the falling man about the origin at time  $t$ : (A)  $dmgt\hat{x}$ ,

(B)  $\frac{1}{2}dmgt\hat{x}$ , (C)  $dmgt\hat{y}$ , (D)  $\frac{1}{2}dmgt\hat{y}$ , (E)  $-\frac{1}{2}dmgt\hat{z}$ .

5. Which wheel is at equilibrium? (A)  (B)  (C)  (D)  (E) 

6. Imagine that a tunnel is drilled from the north pole through the center of the Earth to reach the south pole, and a particle of mass  $m$  is dropped from one end of the tunnel, what is the period of oscillation?

(A)  $\sqrt{\frac{3\pi}{G\rho}}$ , (B)  $\sqrt{\frac{3\pi}{4G\rho}}$ , (C)  $\sqrt{\frac{3}{4\pi G\rho}}$ , (D)  $\sqrt{\frac{3\pi}{2G\rho}}$ , (E)  $\sqrt{\frac{4\pi}{G\rho}}$ .

7. A system of linear simple harmonic oscillation takes place at a frequency of 2 Hz about  $x=0$ . At  $t=0$ , the displacement is  $x(0)=3$  and the velocity  $v(0)=0$ . Find  $v(0.5)$ . (A) 0, (B)  $\pi$ , (C)  $2\pi$ , (D)  $3\pi$ , (E)  $6\pi$ .

8. If the speed and frequency of a sound wave are  $v$  and  $f$  as measured with the source and detector both at rest, then what is the frequency measured by the detector now moving at a speed of  $v_d$  while the sound source is moving in parallel at  $v_s$ ? (A)  $\frac{v+v_d}{v+v_s}f$ , (B)  $\frac{v-v_d}{v-v_s}f$ , (C)  $\frac{v+v_s}{v+v_d}f$ , (D)  $\frac{v-v_d}{v+v_s}f$ , (E)  $\frac{v+v_s}{v-v_d}f$ .

9. Find the heat capacity of two metals, each of masses  $M_1$  and  $M_2$  and specific heat of  $X_1$  and  $X_2$ , when they are connected: (A)  $\frac{X_1}{M_1} + \frac{X_2}{M_2}$ , (B)  $\frac{M_1}{X_1} + \frac{M_2}{X_2}$ , (C)  $M_1X_1 + M_2X_2$ , (D)  $\frac{X_1 + X_2}{M_1 + M_2}$ , (E)  $\frac{M_1 + M_2}{X_1 + X_2}$ .

10. A particle of temperature  $T$  is placed in an environment of temperature  $T_{env}$ , what is the temperature dependence of thermal energy emission from the particle? (A)  $T^4$ , (B)  $T_{env}^4$ , (C)  $T^4 - T_{env}^4$ , (D)  $T$ , (E)  $T_{env}$ .

背面有題

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11. The speed of molecules in an ideal gas follows Maxwell's distribution law. Which of the following is correct when comparing the average speed  $v_a$ , most probable speed  $v_p$ , and root mean square speed  $v_{rms}$ ? (A)  $v_p > v_a > v_{rms}$ , (B)  $v_{rms} > v_a > v_p$ , (C)  $v_{rms} > v_p > v_a$ , (D)  $v_a > v_{rms} > v_p$ , (E)  $v_p > v_{rms} > v_a$ .
12. A point charge  $Q$  is located at a distance of  $R/2$  from the center of a spherical shell of radius  $R$ . If the shell is electrically neutral, what is the net flux of electric field through the sphere shell? (A)  $Q$ , (B)  $\epsilon_0 Q$ , (C)  $Q/\epsilon_0$ , (D) 0, (E)  $Q/2$ .
13. Three point charges are held at the corners of an equilateral triangle of side  $d$ . If  $Q_1 = +q$ ,  $Q_2 = +2q$ ,  $Q_3 = -3q$ , the total electrostatic energy is (A)  $\frac{6q^2}{4\pi\epsilon_0 d}$ , (B)  $\frac{-6q^2}{4\pi\epsilon_0 d}$ , (C)  $\frac{7q^2}{4\pi\epsilon_0 d}$ , (D)  $\frac{-7q^2}{4\pi\epsilon_0 d}$ , (E) 0.
14. An RC circuit consists of a resistor, a capacitor, and an ideal battery of  $emf \mathcal{E}$  in series. On charging, what is the charge in the capacitor at time  $t = RC$  after it starts? (A)  $0.37C\mathcal{E}$ , (B)  $0.5C\mathcal{E}$ , (C)  $0.63C\mathcal{E}$ , (D)  $C\mathcal{E}$ , (E) 0. (Note:  $e = 2.7183$  and  $1/e = 0.3679$ )
15. A long straight wire of radius  $R$  carries a uniformly distributed current  $i$ . What is the magnetic induction at a distance  $r < R$  from the center of the wire? (A) 0, (B)  $\frac{\mu_0 i}{2\pi r}$ , (C)  $\frac{\mu_0 i}{2\pi R}$ , (D)  $\frac{\mu_0 i r}{2\pi R^2}$ , (E)  $\frac{\mu_0 i R}{2\pi r^2}$ .
16. Which of the following describes the Faraday's law? (A)  $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ , (B)  $\oint \vec{B} \cdot d\vec{s} = \mu_0 i$ , (C)  $\oint \vec{B} \cdot d\vec{A} = 0$ , (D)  $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ , (E)  $\oint \vec{E} \cdot d\vec{A} = 0$ .
17. Consider a long solenoid of  $n$  turns per unit length carrying in it a current  $i$ . What is the stored energy density of this solenoid? (A) 0, (B)  $\frac{1}{2} \mu_0 n^2 i^2$ , (C)  $\frac{1}{2} \mu_0 n i^2$ , (D)  $\mu_0 n^2 i^2$ , (E)  $\mu_0 n i^2$
18. What is the angular frequency of resonance for a series RLC circuit? (A)  $\sqrt{RC}$ , (B)  $\sqrt{LC}$ , (C)  $1/\sqrt{RC}$ , (D)  $1/\sqrt{LC}$ , (E)  $\sqrt{\frac{R}{LC}}$ .
19. A beam of unpolarized light is incident onto the interface of material 1 with index of refraction  $n_1$  and material 2 with index of refraction  $n_2$ . What is the angle of incidence at which the reflected light becomes fully polarized? (A)  $\sin^{-1} \frac{n_2}{n_1}$ , (B)  $\sin^{-1} \frac{n_1}{n_2}$ , (C)  $\cos^{-1} \frac{n_1}{n_2}$ , (D)  $\tan^{-1} \frac{n_1}{n_2}$ , (E)  $\tan^{-1} \frac{n_2}{n_1}$ .
20. A plane wave of monochromatic light with wavelength  $\lambda$  is incident normally onto a thin plate cut with a single-slit of width  $a$ . If the first minimum appears at angle  $\theta$ , then the width  $a =$  (A)  $\frac{1}{2} \lambda \sin \theta$ , (B)  $\lambda \sin \theta$ , (C)  $\frac{\lambda}{2 \sin \theta}$ , (D)  $\frac{\lambda}{\sin \theta}$ , (E)  $\frac{3\lambda}{2 \sin \theta}$ .