1. Please explain the following terms:

    (a) [4%] The Pigeonhole Principle

    (b) [4%] Equivalence Class

    (c) [4%] Partially Ordered Set

    (d) [4%] Planar Graph

2. [10%] Determine the number of **positive** integer solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 = 105$$

    where x_i must be exactly divided by 7 for $i = 1, 2, 3, 4$, and 5.

3. Let A be a set and $|A| = n > 0$.

    (a) [5%] How many relations on A are not symmetric?

    (b) [5%] How many relations on A are anti-symmetric?

4. [15%] Find an integer m such that $0 < m < 23 \cdot 29 \cdot 31$ and

$$\begin{cases} m \equiv 1 \pmod{23} \\ m \equiv 0 \pmod{29} \\ m \equiv 2 \pmod{31} \end{cases}$$

    by the Chinese Remainder Theorem.   You should show the details of the steps.

5. Solve the following recurrence relations:

    (a) [10%] $a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \geq 2$, $a_0 = 5$, $a_1 = 12$.

    (b) [10%] $a_{n+1} - 2a_n = 2^n$, $n \geq 0$, $a_0 = 1$.

【背面還有試題】

6. [15%] Let $M = (S, I, O, v, w)$ be a finite state machine, where $S = \{a, b, c, d, e, f\}$ is the set of states; $I = \{0, 1\}$ is the input alphabet for M; $O = \{0, 1\}$ is the output alphabet for M; $v : S \times I \to S$ is the next state function; and $w : S \times I \to O$ is the output function defined as follows:

	v		w	
	0	1	0	1
a	d	c	0	1
b	e	b	1	0
c	b	d	0	0
d	e	c	0	0
e	b	e	1	0
f	a	f	1	0

Which states are equivalent after performing the minimization process on M? Note that you should show the details of the steps.

7. [True/False Questions]

   (a) [2%] If $(F, +, \circ)$ is a field, then it must be an integral domain.

   (b) [2%] Any infinite integral domain $(D, +, \circ)$ is a field.

   (c) [2%] Let $n \in \mathbf{Z}^+$ and $n > 1$. $\mathbf{Z}_n$ is not a field if and only if n is composite.

   (d) [2%] Every group of prime order is acyclic.

   (e) [2%] If G is a finite group of order n with H a subgroup of order m, then $n|m$.

   (f) [2%] Congruence modulo n must be an equivalence relation on $\mathbf{Z}$.

   (g) [2%] If a graph contains a subgraph that is homeomorphic to K_5 or $K_{3,3}$, then the graph is not planar.

1. [10%] Please explain why an algorithm with time complexity $\Theta(n^3)$ may run faster than an algorithm with time complexity $\Theta(n)$, where n is the length of the input.

2. [10%] Please sort 12, 10, 8, 24, and 5 by the heap sort algorithm. You should show the details of the steps.

3. [10%] Please sort 12, 20, 18, 10, and 11 by the quick sort algorithm. You should show the details of the steps.

4. [10%] Seven symbols A, B, C, D, E, F, and G are possibly transmitted on a communication channel where the frequencies of the transmission of these symbols are 8, 9, 10, 12, 18, 20, and 23, respectively. Please design the Huffman codes for the seven symbols and calculate the average code length.

5. (a) [5%] Prove that $\log(n!) = \Omega(n \log n)$.

   (b) [5%] Prove that $\sum_{i=1}^{n} \frac{1}{i} = O(\log n)$.

6. [10%] Let a be an array of integers. Write a **recursive** C function to compute the sum of the elements in the array.

7. [10%] Write a **recursive** C function to find the greatest common divisor (GCD) of two integers

8. [15%] Write a C function for a binary search tree to insert a new key, which does not exist in the tree, into the tree.

9. [15%] Write a C function to reverse a doubly linked circular list so that the last element becomes the first one, and so on. The list is implemented by dynamic variables. It is assumed that the number of elements in the list is greater than 2.

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Problem 1. (20 points)

(1). Explain the major differences between HTTP with non-persistent connections and HTTP with persistent connections.

(2). Explain the similarities and major differences between SMTP and POP3.

(3). What is the major function of DNS? Explain recursive queries in DNS.

(4). Regarding searching for information in a P2P community, what are the major advantages and disadvantages of centralized indexing and query flooding?

Problem 2. (20 points)

(1). What are the major differences between TCP and UDP?

(2). Explain the major difference between packet content error and packet loss. What methods can be used to combat packet content error and packet loss, respectively?

(3). Draw a figure to explain how TCP congestion control algorithm (AIMD) works.

(4). What are the major differences between accumulative acknowledgement and selective repeat?

Problem 3. (20 points)

(1) Explain the differences between packet forwarding and routing in the Internet?

(2) What is the major advantage of dividing IP addresses into classes, compared to using a flat IP address space?

(3) Use an example to explain the exact prefix match and the longest prefix matching for packet forwarding at routers.

(4) Use an example to explain how NAT (Network Address Translation) works.

Problem 4. (20 points)

Write two complete and runnable Java programs. The first Java program is called TCPclient while the second one is called TCPserver. The two Java programs implement the following simple client-server application based on TCP sockets. In particular, the two Java programs interact according to the following order:

(1) The TCPclient reads a line from its standard input (keyboard) and sends the line out its socket to the TCPserver.

(2) The TCPserver reads a line from its connection socket.

(3) The TCPserver converts the line to uppercase.

(4) The TCPserver sends the modified line out its connection socket to the client.

(5) The TCPclient reads the modified line from its socket and prints the line on its standard output (monitor).

Note that the TCPserver always accepts new connections, while the TCPclient terminates once

the modified line has been shown on the monitor. For simplicity, the TCPserver does not support multi-threading. Namely, the TCPserver does not simultaneously support two (or more) clients.

Problem 5. (20 points)

Consider a random access protocol (slotted Aloha) for medium access control in a network. Suppose there are $n \geq 2$ nodes in the network. Time is divided into slots, each is indexed by a natural number. Suppose each node always has packets to send. The probability that a given node transmits a packet in a time slot equals p. The probability that a given node does not transmit packets in a time slot equals $1 - p$.

Let $q_k(n)$ be the probability that k nodes simultaneously transmit packets in a time slot.

(1) (5 points) Suppose $n = 4$ and $p = 0.25$. Derive the probability that at least two nodes simultaneously transmit in time slot two.

(2) (5 points) Let x be a positive real number. When $p = \frac{x}{n}$, calculate $\mu(x) = \lim_{n \to \infty} q_1(n)$.

(3) (10 points) Use Calculus to draw the function of $\mu(x)$ and derive the optimal value of x that maximizes the value of $\mu(x)$.