國立中山大學96學年度轉學生招生考試試題

科目：**微積分【化學系二年級、物理系二年級、電機系二年級、機電系二年級、海工系二年級】**

共十題，每題10分。答題時，每題都必須寫下題號與詳細步驟。請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Suppose that $f(x) = \frac{2+x}{1+x-6x^2}$. Find a closed form for $f^{(n)}(0), n = 1, 2, \ldots$.

2. A ladder 8 m long leans against a wall 4 m high. The lower end of the ladder is pulled away from the wall at a rate of 2 m/sec. How fast is the angle between the top of the ladder and the wall changing when the angle is $60° = \pi/3$ radians?

3. Compute $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{k^2+n^2}$.

4. The base of a certain solid is the circular disk $x^2 + y^2 \leq 4$ in the xy-plane. Each plane perpendicular to the x-axis cuts the solid in an equilateral triangle. Find the volume of the solid.

5. Compute $\int \frac{x+8}{x^2+6x+12}\,dx$.

6. Evaluate
$$\int_0^{\pi/2} \frac{dx}{1+(\tan x)^{\sqrt{2}}}.$$

7. Find the area inside the circle $r = 5\sin\theta$ and outside the limacon $r = 2 + \sin\theta$.

8. Evaluate
$$\lim_{x\to\infty} \left(\frac{1}{x}\frac{a^x - 1}{a - 1}\right)^{1/x}, \quad \text{where } a > 0, a \neq 1.$$

9. Evaluate
$$\frac{1^2}{0!} + \frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \cdots$$

10. Find the volume of the solid bounded by xy-plane, the cylinder $x^2+y^2 = 4$, and the paraboloid $z = 2(x^2 + y^2)$.

一、選擇題，共有二十五題，每題三分

1. Two boys, with masses of 40 kg and 60 kg, respectively, stand on a horizontal frictionless surface holding the ends of a light 10-m long rod. The boys pull themselves together along the rod. When they meet the 40-kg boy will have moved what distance?
   A. 4m B. 5m C. 6m D. 10m
   E. A distance that cannot be known unless the forces the boys exert are given

2. For a wheel spinning with constant angular acceleration on an axis through its center, the ratio of the speed of a point on the rim to the speed of a point halfway between the center and the rim is:
   A. 1 B. 2 C. 1 / 2 D. 4 E. 1 / 4

3. A solid uniform sphere of radius R and mass M has a rotational inertia about a diameter that is given by $(2/5)MR^2$. A light string of length 2R is attached to the surface and used to suspend the sphere from the ceiling. Its rotational inertia about the point of attachment at the ceiling is:
   A. $(2/5)MR^2$ B. $4MR^2$ C. $(7/5)MR^2$ D. $(22/5)MR^2$ E. $(47/5)MR^2$

4. A pulley with radius R and rotational inertia I is free to rotate on a horizontal fixed axis through its center. A string passes over the pulley. A block of mass m_1 is attached to one end and a block of mass m_2 is attached to the other. At one time the block with mass m_1 is moving downward with speed v. If the string does not slip on the pulley, the magnitude of the total angular momentum, about the pulley center, of the blocks and pulley, considered as a system, is given by:
   A. $(m_1 - m_2)vR + Iv / R$ B. $(m_1 + m_2)vR + Iv / R$ C. $(m_1 - m_2)vR - Iv / R$
   D. $(m_1 + m_2)vR - Iv / R$ E. none of the above

5. Two objects with masses of m1 and m2 have the same kinetic energy and are both moving to the right. The same constant force $\vec{F}$ is applied to the left to both masses. If $m_1 = 4m_2$, the ratio of the stopping distance of m_1 to that of m_2 is:
   A. 1:4 B. 4:1 C. 1:2 D. 2:1 E. 1:1

6. A Boston Red Sox baseball player catches a ball of mass m that is moving toward him with speed v. While bringing the ball to rest, his hand moves back a distance d. Assuming constant deceleration, the horizontal force exerted on the ball by his hand is:

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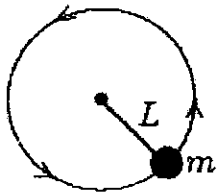
A. mv/d B. mvd C. mv^2/d D. $2mv/d$ E. $mv^2/(2d)$

7. A ball of mass m, at one end of a string of length L, rotates in a vertical circle just fast enough to prevent the string from going slack at the top of the circle. The speed of the ball at the bottom of the circle is:



A. $\sqrt{2gL}$ B. $\sqrt{3gL}$ C. $\sqrt{4gL}$ D. $\sqrt{5gL}$ E. $\sqrt{7gL}$

8. A block of mass m is initially moving to the right on a horizontal frictionless surface at a speed v. It then compresses a spring of spring constant k. At the instant when the kinetic energy of the block is equal to the potential energy of the spring, the spring is compressed a distance of:

A. $v\sqrt{m/2k}$ B. $(1/2)mv^2$ C. $(1/4)mv^2$ D. $mv^2/4k$ E. $(1/4)\sqrt{mv/k}$

9. As a block slides a distance d down an incline, the incline exerts a constant frictional force of magnitude *f* on the block. The quantify *fd* gives the magnitude of:
   A. the work done by the frictional force
   B. the change in the internal energy of the block
   C. the change in the internal energy of the block-incline system
   D. the change in the mechanical energy of the block-Earth system
   E. the change in the kinetic energy of the block

10. A spherical shell has inner radius R_1, outer radius R_2, and mass M, distributed uniformly throughout the shell. The magnitude of the gravitational force exerted on the shell by a point particle of mass m located a distance d from the center, outside the inner radius and inside the outer radius, is:
    A. 0 B. GMm/d^2 C. $GMm/(R_2^3 - d^3)$
    D. $GMm(d^3 - R_1^3)/d^2(R_2^3 - R_1^3)$ E. $GMm/(d^3 - R_1^3)$

11. A 0.25-kg block oscillates on the end of the spring with a spring constant of 200N/m. If the oscillation is started by elongating the spring 0.15m and giving the block a speed of 3.0m/s,then the maximum speed of the block is:

A. 0.13m/s B. 0.18m/s C. 3.7m/s D. 5.2m/s E. 13m/s

12. Two small charged objects attract each other with a force F when separated by a distance d. If the charge on each object is reduced to one-fourth of its original value and the distance between them is reduced to d / 2 the force becomes:
A. F / 16 B. F / 8 C. F / 4 D. F / 2 E. F

13. Positive charge Q is placed on a conducting spherical shell with inner radius R_1 and outer radius R_2. A point charge q is placed at the center of the cavity. The magnitude of the electric field at a point outside the shell, a distance r from the center, is:

A. $Q/4\pi\varepsilon_0 R_1^2$ B. $Q/4\pi\varepsilon_0\left(R_1^2 - r^2\right)$ C. $q/4\pi\varepsilon_0 r^2$

D. $(q+Q)/4\pi\varepsilon_0 r^2$ E. $(q+Q)/4\pi\varepsilon_0\left(R_1^2 - r^2\right)$

14. Positive charge Q is distributed uniformly throughout an insulating sphere of radius R, centered at the origin. A particle with positive charge Q is placed at x = 2R on the x axis. The magnitude of the electric field at x = R / 2 on the x axis is:
A. $Q / 4\pi\varepsilon_0 R^2$ B. $Q / 8\pi\varepsilon_0 R^2$ C. $Q / 72\pi\varepsilon_0 R^2$
D. $17Q / 72\pi\varepsilon_0 R^2$ E. none of these

15. When an external electric field is applied to an insulator with dielectric constant κ, the magnitude of the total electric field in the insulator is proportional to:
A. κ B. 1 /κ C. κ - 1 D. 1 - 1 /κ E. 1 + 1 /κ

16. The capacitance of a spherical capacitor with inner radius a and outer radius b is proportional to:
A. a / b B. b – a C. $b^2 - a^2$ D. ab / (b - a) E. ab / ($b^2 - a^2$)

17. Resistor 1 has twice the resistance of resistor 2. They are connected in parallel to a battery. The ratio of the thermal energy generation rate in 1 to that in 2 is:
A. 1 : 4 B. 1 : 2 C. 1 : 1 D. 2 : 1 E. 4 : 1

18. Electrons (mass m, charge -e) are accelerated from rest through a potential difference V and are then deflected by a magnetic field $\vec{B}$ that is perpendicular to their velocity. The radius of the resulting electron trajectory is:

A. $\left(\sqrt{2eV/m}\right)/B$ B. $B\left(\sqrt{2eV}\right)/m$ C. $\left(\sqrt{2mV/e}\right)/B$

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D. $B\left(\sqrt{2mV}\right)/e$ E. none of these

19. An electron is launched with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The angle θ between $\vec{v}$ and $\vec{B}$ is between 0 and 90°. As a result, the electron follows a helix, its velocity vector $\vec{v}$ returning to its initial value in a time interval of:
A. 2πm / eB B. 2πmv / eB C. 2πmv sinθ / eB
D. 2πmv cosθ / eB E. none of these

20. Two parallel long wires carry the same current and repel each other with a force F per unit length. If both these currents are doubled and the wire separation tripled, the force per unit length becomes:
A. 2F / 9 B. 4F / 9 C. 2F / 3 D. 4F / 3 E. 6F

21. You push a permanent magnet with its north pole away from you toward a loop of conducting wire in front of you. Before the north pole enters the loop the current in the loop is:
A. zero B. clockwise C. counterclockwise D. to your left E. to your right

22. An electron traveling with speed v around a circle of radius r is equivalent to a current of:
A. evr / 2 B. ev / r C. ev / 2πr D. 2πer / v E. 2πev / r

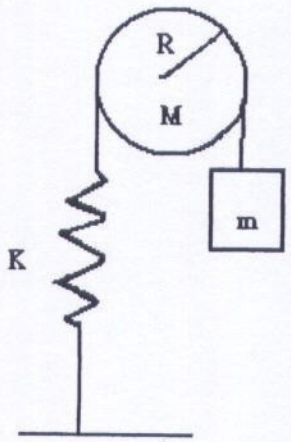
23. The total energy in an LC circuit is 5.0×10^{-6} J. If L = 25 mH the maximum current is:
A. 10mA B. 14mA C. 20mA D. 28mA E. 40mA

24. An RLC series circuit has R = 4Ω, X_C = 3Ω, and X_L = 6Ω. The impedance of this circuit is:
A. 5Ω B. 7Ω C. 9.8Ω D. 13Ω E. 7.8Ω

25. An electromagnetic wave is traveling in the positive x direction with its electric field along the z axis and its magnetic field along the y axis. The fields are related by:
A. $\partial E / \partial x = \mu_0\varepsilon_0 \partial B / \partial x$ B. $\partial E / \partial x = \mu_0\varepsilon_0 \partial B / \partial t$ C. $\partial B / \partial x = \mu_0\varepsilon_0 \partial E / \partial x$
D. $\partial B / \partial x = \mu\varepsilon_0 \partial E / \partial t$ E. $\partial B / \partial x = -\mu_0\varepsilon_0 \partial E / \partial t$

二、計算題，共二十五分

1. A block of mass m is attached to a vertical spring via a string that hangs over a pulley ($\mathbf{I}=\mathbf{MR}^2/2$) of mass **M** and radius **R**, The string doesn't slip. Find the angular frequency of oscillation when the block is pulled from the equilibrium position downward and released. (10%)



2. A capacitor consists of two long concentric metal cylinders of length **L** with the line charge density λ. The inner and outer cylinders have radii **a** and **b**, respectively.
   (a) Find the capacitance in this cylindrical capacitor.
   (b) Find the energy stored in this cylindrical capacitor in terms of line charge density. (15%)