國立中山大學九十一學年度轉學生招生考試試題

科目：**微積分**【物理系二年級、電機系二年級、資工系二年級、海工系二年級】

共七題，滿分100分。答題時，每題都必須寫下題號與步驟。

1. Let $a_i > 0$ for $i = 1, 2 \ldots, n$. Find
   (a) $\lim_{p \to 0} (a_1^p + a_2^p + \cdots + a_n^p)^{1/p}$. (5分)
   (b) $\lim_{p \to -\infty} (a_1^p + a_2^p + \cdots + a_n^p)^{1/p}$. (5分)
   (c) $\lim_{p \to \infty} (a_1^p + a_2^p + \cdots + a_n^p)^{1/p}$. (5分)

2. Let $\alpha = \sqrt{2} + \sqrt{3}$.
   (a) Find a polynomial $p(x)$ with integer coefficients such that $p(\alpha) = 0$. (5分)
   (b) Use one iterate of Newton method with initial value 3 to approximate α. (10分)

3. Calculate $\int \frac{1 + x^3 + x^4 + x^5}{1 + x^2 + x^4 + x^6}\, dx$. (10分)

4. Let $L = \{(r, \theta) \,|\, r = 1 + \sin\theta, \theta \in R\}$.
   (a) Sketch the curve L. (5分)
   (b) Calculate the area of the region enclosed by L. (10分)

5. Let $p(x) = x^{100} + 1$ and $g(x) = (x - 1)^2$. Find the remainder of $p(x)/g(x)$. (10分)

6. Find the radius of convergence and interval of convergence of the power series
$$\sum_{k=0}^{\infty} 2^k (x - 4)^k / \ln(k + 2).$$
(15分)

7. Let $S = \{(x, y, z) \,|\, 0 \le \sqrt{x^2 + y^2} \le 4 - z,\ 0 \le z \le 2\}$.
   (a) Sketch the graph of S. (5分)
   (b) Find the volume of S. (10分)
   (c) Find the tangent plane to S at $(3, 0, 1)$. (5分)

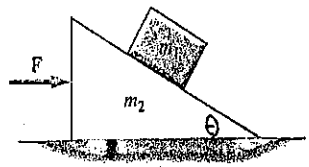
～全卷完～

國立中山大學九十一學年度轉學生招生考試試題

科目：普通物理【物理系二年級、電機系二年級】

(10% for each problem)

1. A rectangular block of mass m_1 rests on a wedge-shaped block of m_2, as shown in the following figure. All contact surfaces are frictionless. Find an expression for the magnitude of the horizontal force F that must be applied to the wedge in order that the rectangular block not slide along the wedge.



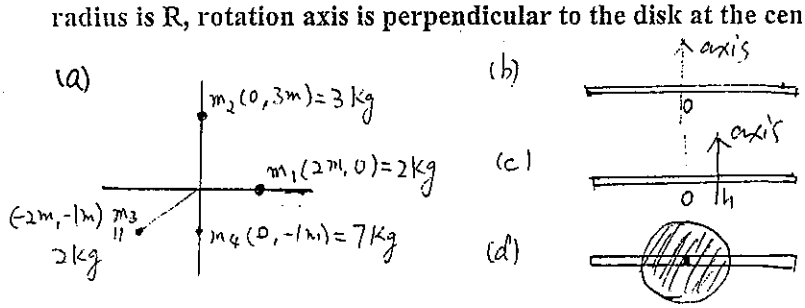
2. What are the moment of inertia I of the following configurations.

(a) several masses rotate around the z-axis.

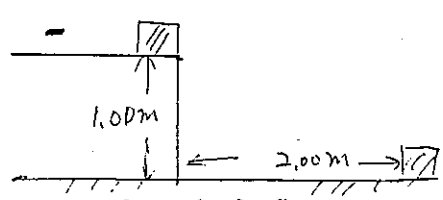
(b) a uniform long rod, length L, mass m, rotation axis perpendicular to the rod through center.

(c) the above rod, but the rotation axis moves h away from center.

(d) a combination structure, the center part is a uniform disk, mass is M, radius is R, rotation axis is perpendicular to the disk at the center.

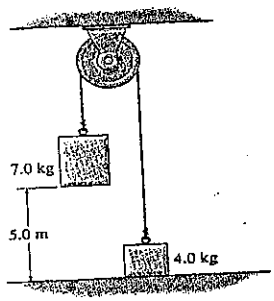


3. An 8.00-g bullet is fired into a 2.50-kg block that is initially at rest at the edge of a frictionless table of height 1.00 m (Fig.). The bullet remains in the block, and after impact the block lands 2.00 m from the bottom of the table. Determine the initial speed of the bullet.

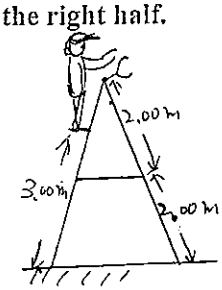


4. The masses shown in the figure are connected by a massless string over a frictionless pulley and are released from rest. Use energy conservation to find (a) the velocity of the 7.0-kg mass just before it hits the floor, (b) the

maximum height reached by the 4.0-kg mass, and (c) the fraction of the system's initial mechanical energy lost when the 7.0-kg mass comes to rest on the floor.



5. A stepladder of negligible weight is constructed as shown in the figure. A painter with a mass of 70.0 kg stands on the ladder 3.00 m from the bottom. Assuming that the floor is frictionless, find (a) the tension in the horizontal bar connecting the two halves of the ladder, (b) the normal forces at the single hinge C that the left half of the ladder exerts on the right half.



6. The motion of a particle is described by

$$x=(45\text{ cm})[\sin(\pi t+\pi/6)],$$

with x in cm and t in seconds. At what time is the potential energy twice the kinetic energy? What is the position of the particle at this time?

7. What is the Hall effect? How do we use it to measure the nature and density of charge carriers? How do we use it to measure the magnetic field strength?

8. Explain the electrical conductivity properties of conductors, insulators, and semiconductors (including intrinsic, N-type, and P-type semiconductors) with band theory.

9. What are the major disagreements with the classical predictions in the photoelectric effect experiments? And how does Einstein explain the photoelectric effect?

10. Compare the LC circuits and the mass-spring systems. Offer the mathematical and physical explanation of the energy exchange between capacitor and inductor.