Do all problems in detail.

(1)

(a) [10%] Evaluate $\int_0^\infty e^{-st}\sin at\,dt$ for $a, s > 0$.

(b) [10%] Evaluate $\int_0^2 x^3\sqrt{4-x^2}\,dx$.

(2)

(a) [15%] Determine whether the series $\sum_{n=1}^\infty \frac{n(n+1)}{(n+2)^3}$ is convergent or divergent.

(b) [10%] Find the radius of convergence of the power series $\sum_{n=1}^\infty \frac{x^n}{n^2+1}$.

(3)

(a) [10%] Find all local extrema of $f(x,y) = x^2 - xy + y^2 + 2x + 2y + 1$.

(b) [10%] Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{x^3}{\sqrt{x^2+y^2}}\,dz\,dy\,dx$.

(4)

(a) [10%] Is the vector field $\mathrm{f}(x,y) = (x^2 + xy^2, x^2y + y^2)$ conservative?

(b) [15%] A particle is moving along the parabola $y = x^2 + x - 1$ subject to the vector field given as in (a). Find the work done in moving from the point $(-1,-1)$ to the point $(3,11)$ if forces are measured in newtons and distances are measured in meters?

(c) [10%] Evaluate the surface integral $\int\int_S (x - 2x^3 + 3x^5)\,d\sigma$ where S is the part of the surface $z = x^2$ lying over $\{(x,y) : -1 \le x \le 1, 0 \le y \le 2\}$.

1. A uniform stick of length L and mass M is held vertically with one end resting on the floor. When the stick is released, it rotates around its lower end until it hits the floor. Assuming that the lower end of the stick does not slip, what is the linear velocity of its upper end when it hits the floor? (10%)
2. A bowling ball is thrown straight down the alley. When it starts, its center of mass has a speed of v_0 and it is sliding without rotating. Determine how far the ball moves down the alley before it starts rolling without slipping. Exprss your answer in terms of v_0, g, and μ_K. (15%)
3. A device known as a Venturi meter (Fig.1) is used to measure the speed of a fluid flowing through a pipe. Show that the speed of the fluid in the pipe is $v_1 = A_2\sqrt{\dfrac{2(\rho_{Hg}-\rho_f)gh}{\rho_f(A_1^2-A_2^2)}}$ (10%)
4. A rocket whose initial mass including fuel m_0 is fired vertically upward from the earth's surface. Fuel is ejected out the rear at a constant relative speed u and a rate dm/dt = -r, where r is positive. What is the velocity of the rocket as a function of time during the period that fuel is being burned (neglect the air resistance) ? (15%)
5. A long, straight wire of radius 1.0 mm is charged uniformly at $-5.0\times10^{-8}C/m$. The wire is surrounded by a cylindrical conducting shell of inner radius 5.0 mm. If an electron is released at the wire, what is its speed when it reaches the shell(Fig.2)? (10%)
6. Charge is distributed throughout a spherical volume of radius R with a density $\rho = br^2$, where b is a constant. Determine the electric field due to the charge at points both inside and outside the sphere. (10%)
7. A current I flows around a wire bent into the shape of a square of side a. What is the magnetic field at the point P that is a distance z above the center of square. (10%)
8. A coil of 1000 turns encloses an area of 25 cm^2. It is rotated in 0.010 s from a position where its plane is perpendicular to the earth's magnetic field to one where its plane is parallel to the field. If the strength of the field is 6.0×10^{-5}T, what is the average emf induced in the coil? (10%)
9. An ac generator whose emf is given by $v(t) = (4.0V)\sin(1.0\times10^4 t)$ is connected to an RLC circuit for which $L = 2.0\times10^{-3}H, C = 4.0\times10^{-6}F, R = 5.0\Omega$. Find (a) the phase difference between the current and the emf and (b) the average power output of the generator? (10%)

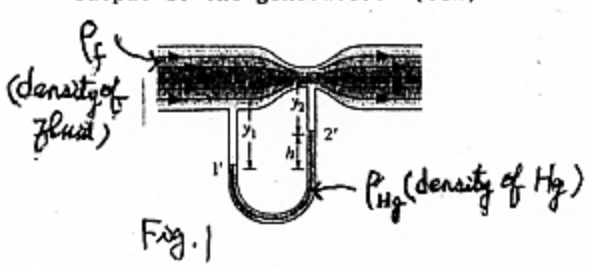

Fig.1


Fig.2