

1. 四尖內擺線 $x^{2/3} + y^{2/3} = 4$ 之任一切線在兩軸之間有定長 h , 求 h 之值.

2. 試決定方程式 $x^3 - x + b = 0$ 之實根個數.

3. 求 $\int \frac{\sqrt[3]{\sin x}}{\cos x} dx$

4. 求 $\int_0^1 e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$

5. 求擺線 $x = a(t - \sin t)$, $y = a(1 - \cos t)$ 一拱繞 x 軸旋轉所得旋轉體之體積.

6. 求曲面 $xy = z$ 及 $z = 0$ 所夾部份之體積.

7. 討論級數 $\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt{k+10}}$ 之斂散性.

8. 求球面 $x^2 + y^2 + z^2 = 1$ 上距離點 $(1, 2, 3)$ 最近之點.

9. 求曲面 $\sqrt{x^2 + y^2} + z = 2$ 與兩個平面 $x = z$ 及 $x = 0$ 所圍部份之體積.

10. 求函數 $\sin^4 x$ 對 $x = 0$ 之冪級數展開式.

1. A ball is thrown horizontally from the top of a building 35.0 m high. The ball strikes the ground at a point 80.0 m from the base of the building. Find (a) the time the ball is in flight, (3%)(b) its initial velocity, (3%) and (c) the x and y components of velocity just before the ball strikes the ground (4%).

2. A mass $M = 2.20$ kg is accelerated across a rough surface by a rope passing over a pulley, as shown in Fig. 1. The tension in the rope is 10.0 N and the pulley is 10.0 cm above the top of the block. The coefficient of kinetic friction is 0.400 . (a) Determine the acceleration of the block when $x = 40.0$ cm. (5%) (b) Find the value of x at which the acceleration becomes zero (5%).

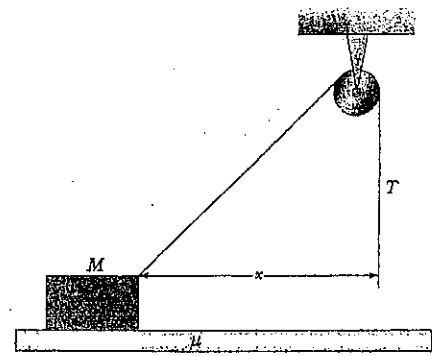


Fig. 1

3. A bullet of mass m_1 is fired into a wooden block of mass m_2 initially at rest on a horizontal surface. After impact, the block slides a distance d before coming to rest. If the coefficient of friction between block and surface is μ , what was the speed of the bullet immediately before impact? (10%)

4. A particle of mass m slides inside a frictionless hemispherical bowl of radius R . Show that if it starts from rest with a small displacement from equilibrium, the particle moves in simple harmonic motion with an angular frequency equal to that of a simple pendulum of length R . That is, $\omega = (g/R)^{0.5}$. (10%)

5. One mole of gas initially at a pressure of 2.00 atm and a volume of 0.300 L has an internal energy that may be set equal to 91.0 J. In its final state, the pressure is 1.50 atm, the volume is 0.800 L, and the internal energy equals 182 J. For the three paths IAF, IBF, and IF in Fig. 2, calculate (a) the work done by the gas (5%) and (b) the net thermal energy transferred in the process (5%). (1.00 atm = 1.013×10^5 Pa)

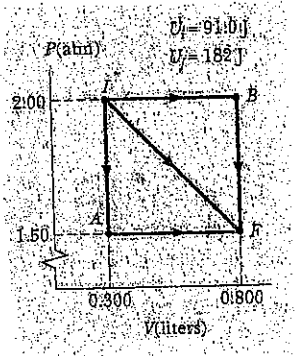


Fig. 2

6. A continuous line of charge lies along the x axis, extending from $x = +x_0$ to positive infinity. The line carries a uniform linear charge density λ_0 . What are the magnitude and direction of the electric field at the origin? (10%)

7. A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ that is concentric with a smaller conducting sphere of radius a and charge $+Q$ (Fig.3). Show that its capacitance is (10%)

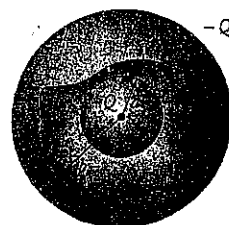


Fig.3

$$C = \frac{ab}{k_e(b-a)}$$

8. The ammeter in Fig.4 reads 2.00 A. Find I_1 , I_2 , and ϵ . (10%)

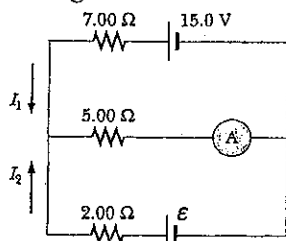


Fig.4

9. A conducting rectangular loop of mass M , resistance R , and dimensions w by l falls from rest into a magnetic field B , as shown in Fig.5. The loop accelerates until it reaches a terminal speed v_t . Show that (10%)

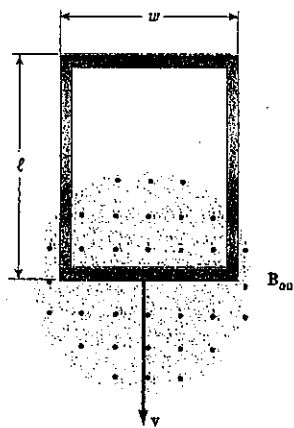


Fig.5

$$v_t = \frac{MgR}{B^2 w^2}$$

10. An air wedge is formed between two glass plates separated at one edge by a very fine wire as in Fig.6. When the wedge is illuminated from above by 600-nm light, 30 dark fringe are observed. Calculate the radius of the wire.(10%)



Fig.6