

國立中山大學 103 學年度轉學考招生考試試題

科目名稱：微積分【電機系二年級】

題號：731001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）

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微積分

1. (30%) 請回答下列是非題，只須回答是(O)或不是(X)，不用額外說明原因。

(a). () $\lim_{x \rightarrow -1} \frac{1}{1+x}$ 之極限值不存在。

(b). () 若函數 $g(x) = \int_{-1}^x \cos(\pi t^2) dt$ ，則 $g'(-1) = 0$ 。

(c). () 對於某一函數 f 與任意 a 值，若 $\lim_{x \rightarrow a^+} f(x)$ 且 $\lim_{x \rightarrow a^-} f(x)$ 皆相等且存在，則 f 在 $x = a$ 點連續。

(d). () 對於任何奇函數(odd function)，若 $f(0) = \lim_{x \rightarrow 0^+} f(x)$ ，則函數 f 在 $x = 0$ 處連續。

(e). () $\sum_{n=1}^{\infty} \frac{n^{1/2} + 5}{n}$ 為一個收斂數列。

(f). () 對於任意正數 ε 與任意 x ，當 $0 < |x - 2| < \sqrt{\varepsilon}$ ，則 $|(x^2 - 4x + 9) - 5| < \varepsilon$ 。

2. (20%)

(a). (5%) 何謂均值定理(Mean Value Theorem)?

(b). (10%) 假設一函數 $f(x) = x - e \ln x$ ，證明若 $b > e$ ，則 $f(b) > 0$ 。
(暗示：利用均值定理)

(c). (5%) 利用上述(b)之結果，說明 e^π 與 π^e 何者較大。

3. (20%) 計算下列極限值。

(a). (10%) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - x)$ 。

(b). (10%) $\lim_{x \rightarrow -\infty} x e^x$ 。

4. (20%) 計算下列微分。

(a). (10%) 給定 $y = \int_3^{x^2} \arctan(t) dt$ ，求 $\frac{dy}{dx}$ 。

(b). (10%) 給定 $f(x) = x + \cos x$ ，求 $\frac{df^{-1}(x)}{dx}$ 在 $x = 1$ 之值。(暗示： $y = f^{-1}(x)$ 會滿足 $x = y + \cos y$)。

[背面有試題]

背面有題

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5. (10%) 計算下列曲線弧度之長度

$$\left(2t+3, \frac{4}{3}t^{3/2}+3, \frac{1}{2}t^2+4 \right)$$

其中 t 的範圍為 $1 \leq t \leq 2$ 。

背面有題

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科目名稱：普通物理【電機系二年級】

題號：731002

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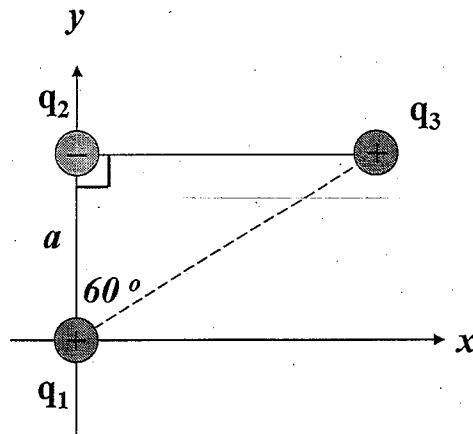
共 2 頁第 1 頁

1. Consider three point charges located at the corners of a right triangle.

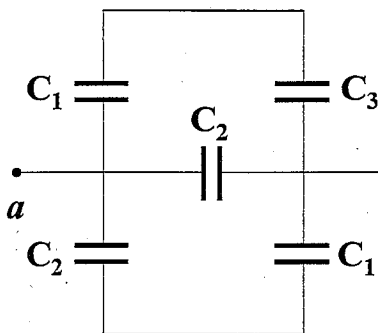
$q_1 = 7 \mu\text{C}$, $q_2 = -5 \mu\text{C}$, $q_3 = 4 \mu\text{C}$, and $a = 0.15 \text{ m}$.

The permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$.

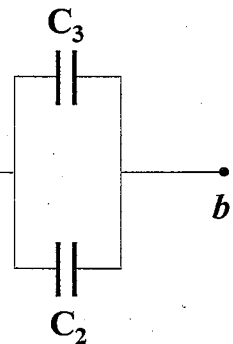
Find the resultant force exerted on q_3 .



2. Find the equivalent capacitance between points a and b in the combination of capacitors shown below if $C_1 = 5 \mu\text{F}$, $C_2 = 8 \mu\text{F}$, and $C_3 = 3 \mu\text{F}$.



(a)

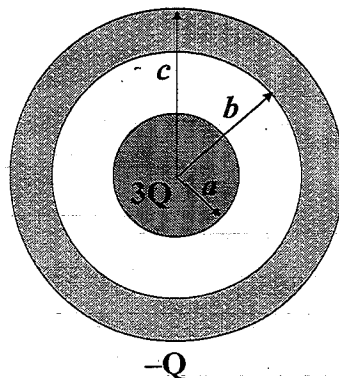


(b)

(10%)

3. A solid conducting sphere of a radius a carries a net positive charge $3Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-Q$. Using Gauss's law, find the electric field in the regions:

- (a) $r < a$
- (b) $a < r < b$
- (c) $b < r < c$
- (d) $r > c$



(10%)

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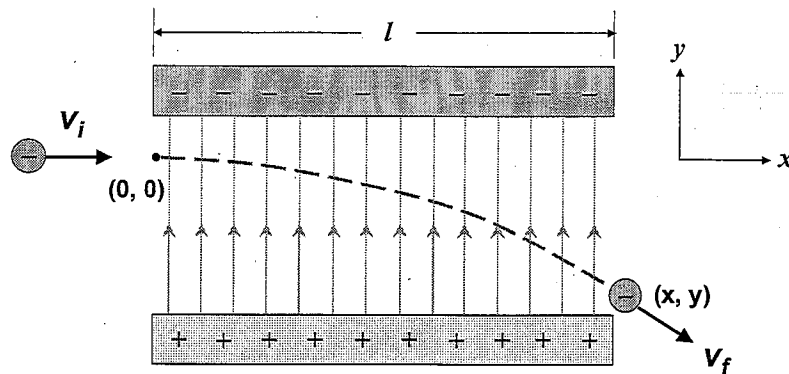
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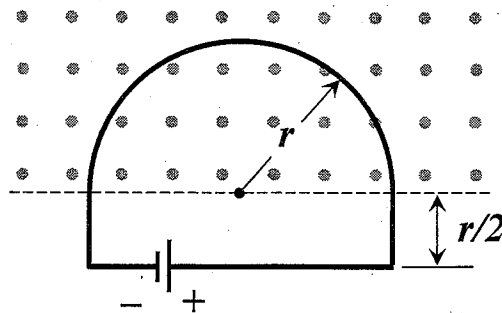
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4. An electron enters the region of a uniform electric field with $v_i = 4 \times 10^6$ m/s and $E = 300$ N/C. The horizontal length of the plates is $l = 0.3$ m. The mass of the electron is $m = 9.11 \times 10^{-31}$ kg.
- Find the acceleration of the electron while it is in the electric field.
 - Find the time it takes the electron to travel through the field.
 - What is the vertical displacement y of the electron while it just left the field?
 - Find the speed v_f of the electron while it just left the field.



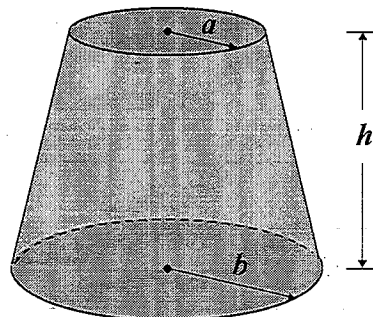
(20%)

5. A conducting loop consisting of a half-circle of radius $r = 0.35$ m and three straight sections. The half-circle lies in a uniform magnetic field \vec{B} that is directed out of the paper. The field magnitude is given by $B = 2t^2 + 7t + 6$, with B in teslas and t in seconds. An ideal battery with emf = 2 V is connected to the loop. The resistance of the loop is 3Ω .
- What is the magnitude and direction of the emf induced around the loop by field \vec{B} at $t = 10$ s?
 - What is the current in the loop at $t = 10$ s?



(20%)

6. Consider a material of resistivity ρ in a shape of a truncated cone of altitude h , and radii a and b , for the top and the bottom ends, respectively. Assuming that the current is distributed uniformly throughout the cross-section of the cone, what is the resistance between the two ends?



(20%)

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