國立中山大學101學年度轉學生招生考試試題

科目：微積分【電機系二年級】

題號：7024

1. (20%) 決定下列之數列發散或收斂，並找出其極限值。

(a). (5%) $a_n=\sin^2\frac{n}{2n+1}+\cos^2\frac{n}{2n+1}$。

(b). (5%) $a_n=1+\frac{\left(\cos\frac{\pi}{2}\right)^n}{n}$。

(c). (5%) $a_n=n\sin\left(\frac{1}{n}\right)$。

(d). (5%) $a_n=\frac{e^n}{n^4}$。

2.(10%) 計算 $\lim_{n\to\infty}\frac{1^6+2^6+\cdots+n^6}{n^7}$。

3.(15%) 計算下列微分。

(a). (5%) 給定 $x^{1/3}+y^{2/3}=3$，求 $\frac{dy}{dx}=?$

(b). (5%) 給定 $y=3^{\ln x}$，求 $\frac{dy}{dx}=?$

(c). (5%) 給定 $y=\ln 3^x$，求 $\frac{dy}{dx}=?$

4.(5%) 計算 $\int_0^{\pi}\left(\sin x+|\cos x|\right)dx=?$

5.(20%) 求下列積分值。

(a). (10%) $\int_{-1}^{1}\int_{|y|}^{1}\left(x^2+y\right)dxdy$。

(b). (10%) $\iiint_W z^2dxdydz$，其中 W 是由平面 $x=0,y=0,z=0$ 與 $x=0,y=0,z=1$ 及圓柱 $x^2+y^2=1,\ x,y\ge 0$ 所構成的封閉區域。

6. (15%) 計算 $\int_0^1 e^{x^{1/3}}dx=?$

7.(15 %) 假設 $(x(t),y(t))$，$0\le t\le 2$，為一個平面內的路徑，$f(x,y)$ 為 x 與 y 的一階可微函數。假設 f_x 與 f_y 分別為 x 與 y 的偏導數且 $\left(\frac{dx}{dt}\right)f_x+\left(\frac{dy}{dt}\right)f_y\le 0$，證明：

$$f.(x(2),y(2))\le f\left(x(0),y(0)\right).$$

國立中山大學101學年度轉學生招生考試試題

科目：普通物理【電機系學士班二年級】

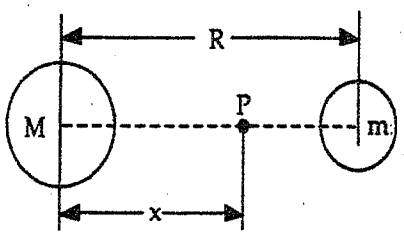
題號：7025

一.單選題 (每題三分;共25題)

1. An 800-N passenger in a car presses against the car door with a 200N force when the car makes a left turn at 13 m/s. The door will pop open under a force of 800N. Of the following, the least speed for which the passenger is thrown out of the car is: (A) 14 m/s (B) 19 m/s (C) 20 m/s (D) 26 m/s (E) 54 m/s.

2. A 1000 kg cart is rolling to the right at 5.0 m/s. A 70 kg man is standing on the right end of the cart. For an unknown reason, he suddenly runs to the left with a speed of 10 m/s relative to the cart. What is the speed of the cart after the man starts running? (A) 8.65 m/s (B) 4.65 m/s (C)7.65 m/s (D) 6.65 m/s (E) 5.65 m/s.

3. Two planets have masses M and m, and the ratio $M/m = 25$. The distance between the planets is R. The point P is between the planets as shown, and the distance between M and P is x. At P, the gravitational force on an object due to M and m are equal in magnitude. The value of x is (A) $5R/6$ (B) $25R/36$ (C) $R/25$ (D) $6R/5$ (E) none of these.



4. A satellite moves in a circular orbit just above the surface of a planet, assumed to offer no air resistance. If the satellite's orbit speed is v, the escape velocity from the planet is (A) v (B) $\sqrt{2}v$ (C) $\sqrt{3}v$ (D) $2v$ (E) $2\sqrt{2}v$.

5. An object of mass m is observed to move in a straight line with velocity given by $v = bt^2 - ct$, where b and c are positive constants. The expression for the force on the object as a function of time is (A) $mbt^2 - mct$ (B) $2mbt^2 - 2mct$ (C) $2mbt - mc$ (D) $mbt - mc$ (E) $mbt^2 - 0.5mct$.

6. A rotating wheel 80 cm in diameter is decelerating at 0.21 rad/s^2. What should be the initial angular speed if the wheel is to stop after exactly one revolution? (A) 0.8 rad/s (B) 0.9 rad/s (C) 1.2 rad/s (D) 1.4 rad/s (E)1.6 rad/s.

7. A particle is acted on by a force of the form $\vec{F} = 2xy\hat{i} + 3xy^2\hat{j}$ (N). If the particle moves along the path (1m,1m) → (3m,1m) → (3m,3m), what is the work done by the force during the movement? (A) 48J (B) 56J (C) 66J (D) 76J (E) 86J.

8. A 0.5 kg block attached to a spring is pulled a distance of 20 cm horizontally and released. If the period of the oscillation is 0.8 s, what is the kinetic energy of the block at t = 0.2 s? (A) 0.42J (B) 0.62J (C) 0.82J (D) 1.02J (E) 1.24J.

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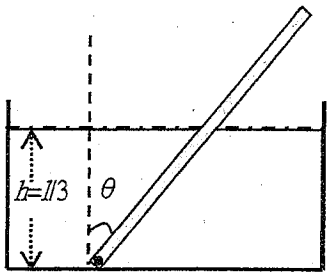
9. A wooden rod of uniform cross section and of length l is hinged at the bottom of a tank which is filled with water to a height $h = l/3$. If the density of wood is 0.45 g/cm^3, find the angle θ from the vertical at which the rod is in equilibrium.(A) $\theta = 60°$ (B) $\theta = 30°$ (C) $\theta = 45°$ (D) $\theta = 37°$ (E) $\theta = 53°$



10. A cube of side $2a$ and mass M is sliding on a frictionless surface with uniform velocity v_0, as shown in the Figure. It hits a small obstacle at the end of the table, which causes the cube to tilt as in Fig. b. Find the minimum value of v_0 such that the cube will fall off the table (g is the gravitational acceleration constant on earth). (A) $\sqrt{2ag(\sqrt{2}-1)}$ (B) $\sqrt{4ag(\sqrt{2}-1)}$ (C) $\sqrt{\frac{16}{3}ag(\sqrt{2}-1)}$ (D) $\sqrt{\frac{4}{3}ag(\sqrt{2}-1)}$ (E) $\sqrt{\frac{8}{3}ag(\sqrt{2}-1)}$.

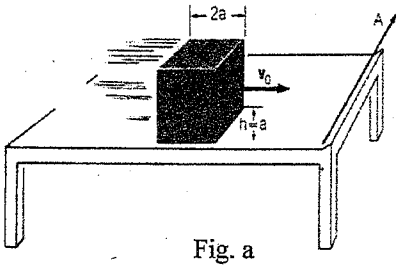


Fig. a

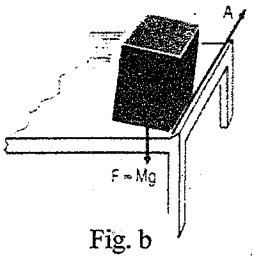


Fig. b

11. Two ideal gases, each consisting of N monatomic molecules, are in thermal equilibrium with each other and equilibrium is maintained as the temperature is increased. A molecule of the first gas has mass m and a molecule of the second has mass $4m$. The ratio of the changes in the internal energies $\Delta E_{4m}/\Delta E_m$ is: (A) 1/4 (B) 1/2 (C) 1 (D) 2 (E) 4.

12. A heat engine operates between 200K and 100K. In each cycle it takes 100J from the hot reservoir, loses 25J to the cold reservoir and does 75J of work. This heat engine violates : (A) both the first and second laws of thermodynamics (B) the first law but not the second law of thermodynamics (C) the second law but not the first law of thermodynamics (D) neither the first law nor the second law of thermodynamics (E) cannot answer without knowing the mechanical equivalent of heat.

13. $TV^{\gamma-1}$ is constant for an ideal gas undergoing an adiabatic process, where γ is the ratio of heat capacities C_P / C_V. This is a direct consequence of (A) the zeroth law of thermodynamics alone (B) the zeroth law and the ideal gas equation of state (C) the first law of thermodynamics alone (D) the ideal gas equation of state alone (E) the first law and the ideal gas equation of state.

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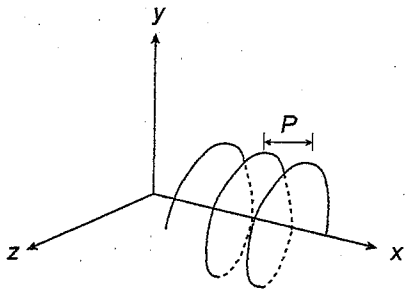
14. The temperatures T_C of the cold reservoirs and the temperatures T_H of the hot reservoirs for four Carnot heat engines are
engine 1: $T_C = 400\text{K}$ and $T_H = 500\text{K}$
engine 2: $T_C = 500\text{K}$ and $T_H = 600\text{K}$
engine 3: $T_C = 400\text{K}$ and $T_H = 600\text{K}$
engine 4: $T_C = 600\text{K}$ and $T_H = 800\text{K}$
Rank these engines according to their efficiencies, least to greatest
(A) 1, 2, 3, 4 (B) 1 and 2 tie, then 3 and 4 tie (C) 2, 1, 3, 4 (D) 1, 2, 4, 3 (E) 2, 1, 4, 3

15. A charged particle ($m = 2.0$ g, $q = -50\ \mu$C) moves in a region of uniform field along a helical path (radius = 4.0 cm, pitch = 8.0 cm) as shown. What is the angle between the velocity of the particle and the magnetic field?
(A)27° (B) 72° (C) 63° (D) 18° (E) 58°

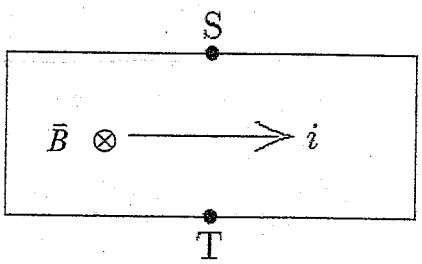


16. Two parallel wires, 4cm apart, carry currents of 2A and 4A, respectively, in the same direction. The force per unit length in N/m of one wire on the other is (A) 1.0×10^{-3}, repulsive (B) 1.0×10^{-3}, attractive (C) 4.0×10^{-5}, repulsive (D) 4.0×10^{-5}, attractive (E) none of these.

17. A 30-μF capacitor is charged to 40 V and then connected across an initially uncharged 20-μF capacitor. What is the final potential difference across the 30-μF capacitor?
(A) 15 V (B) 24 V (C) 18 V (D) 21 V (E) 40 V

18. Two conducting spheres are far apart. The smaller sphere carries a total charge Q. The larger sphere has a radius that is twice that of the smaller and is neutral. After the two spheres are connected by a conducting wire, the charges on the smaller and larger spheres, respectively, are: (A) $Q/2$ and $Q/2$ (B) $Q/3$ and $2Q/3$ (C) $2Q/3$ and $Q/3$ (D) zero and Q (E) $2Q$ and $-Q$

19. The current is from left to right in the conductor shown. The magnetic field $\vec{B}$ is into the page and point S is at a higher potential than point T. The charge carriers are (A) positive (B) negative (C) neutral (D) absent (E) moving near the speed of light



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20. The semicircular wire of radius R connects two straight wire segments. If a current I follows along the wire, the magnetic field at the center of the semicircular wire due to the current in the semicircular wire is (A) $\mu_0 I/(2R)$ (B) $\mu_0 I/(4R)$ (C) $\mu_0 I/(8R)$ (D) $\mu_0 I/(2\pi R)$ (E) $\mu_0 I/(4\pi R)$.

21. A proton is accelerated from rest through a potential difference of 2.5 kV and then moves perpendicularly through a uniform 0.60-T magnetic field. What is the radius of the resulting path?
(A) 15 mm (B) 12 mm (C) 18 mm (D) 24 mm (E) 8.5 mm

22. For a charge moving in the x direction, if $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{y}$ the total force is (A) in the x direction (B) in the z direction (C) on the x-y plane (D) on the z axis (E) on the x-z plane.

23. The free-fall acceleration on the surface of the Moon is about one-sixth that on the surface of the Earth. If the radius of the Moon is about $0.250R_E$, the ratio of their average densities, ρ_{Moon}/ρ_{Earth}, is (A) 2/3 (B) 1/2 (C) 1/3 (D) 1/6 (E) 1/4.

24. A 2-μF capacitor in series with a 2-k resistor is connected to a 60-Hz ac source. Calculate the impedance of the circuit.
(A) 1500 ohms (B) 1800 ohms (C) 2100 ohms (D) 2400 ohms (E) 8600 ohms

25. A conductor of radius *r*, length and resistivity has resistance *R*. It is melted down and formed into a new conductor, also cylindrical, with one fourth the length of the original conductor. The resistance of the new conductor is (A) $\frac{1}{16}$R (B) $\frac{1}{4}$R (C) R (D) 4R (E) 16R

二.複選題 (每題五分;共 5 題)

26. A man and a boy, initially at rest on frictionless ice, push each other apart. After a short time, (A) the man is further from the starting point. (B) the boy is further from the starting point. (C) neither is further from the starting point. (D) the man is moving with a higher speed. (E) the boy is moving with a higher speed.

27. A particle moves as following as a function of time: $X = 3.0m \times \cos(2.0 radian/s \times T + \pi/3)$, where distance (X) is measured in meters (m) and time (T) in seconds (s). Which answers are correct in the following? (A) The amplitude of the simple harmonic motion is 3.0 m. (B) The amplitude of the simple harmonic motion is 6.0 m. (C) The angular frequency is 2.0 radian/s. (D) The angular frequency is 0.318 radian/s. (E) The angular frequency is 6.28 radian/s.

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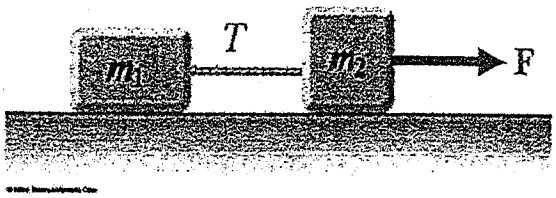
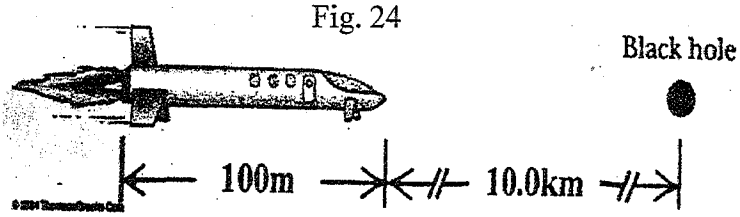
Fig. 23



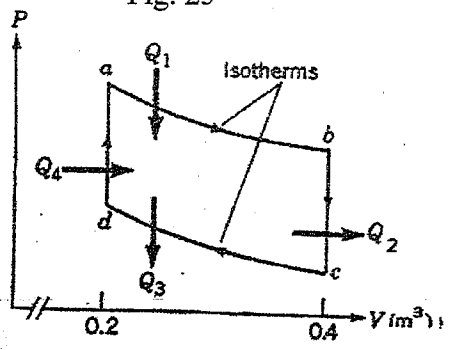
28. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force F as in Fig. 21. Suppose that $F = 68.0N$, $m_1 = 12.0kg$, $m_2 = 18.0kg$, and the coefficient of kinetic friction between each block and the surface is 0.100. Which answers are correct in the following? (A) The magnitude of the acceleration of the system is 3.20 m/s^2. (B) The magnitude of the acceleration of the system is 2.58 m/s^2. (C) The magnitude of the acceleration of the system is 1.29 m/s^2. (D) The tension T of the system is 13. 6N. (E) The tension T of the system is 27.2N.

Fig. 24



29. A spacecraft in the shape of a long cylinder has a length of 100m and its mass with occupants is 1000kg. It has strayed too close to a black hole having a mass 100 times that of the Sun. The nose of the spacecraft points toward the black hole, and the distance between the nose and the craft of the black hole is 10.0km. Which answers are correct in the following? (A) The total force on the spacecraft is 3.93×10^{17} N. (B) The total force on the spacecraft is 2.62×10^{17}N. (C) The total force on the spacecraft is 1.31×10^{17}N. (D) The difference in the gravitational fields acting on the occupants in the nose of the ship and on those in the rear of the ship, farthest from the black hole, is 2.62×10^{12}N/kg. (E) The difference in the gravitational fields acting on the occupants in the nose of the ship and on those in the rear of the ship, farthest from the black hole, is 1.31×10^{12}N/kg.

Fig. 25



30. One mole of ideal monatomic gas is taken around the reversible cycle of Fig. 25. The isothermals are at 500K and 300K. (R is the gas constant

(A) $Q_1 = 500R\ln 2$,

(B) $Q_2 = -300R$,

(C) $Q_3 = -300R\ln 2$,

(D) the work done is one cycle is $200R\ln 2$,

(E) the efficiency of the engine is 0.214.