國立中山大學100學年度學士班轉學生招生考試試題

科目：微積分【電機工程學系二年級】

共十題，每題 10 分。答題時，每題都必須寫下題號與詳細步驟。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Given $x^2y^2 - 2x = 3$, find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of x and y.

2. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r.

3. Find the volume of the solid generated by revolving the plane region $y = 4x - x^2$, $y = 0$, about the line $x = 5$.

4. Find $\lim_{x\to 0^+}(e^x + x)^{2/x}$.

5. Find $\int x^7 \ln x \, \mathrm{d}x$.

6. Is the following series convergent or divergent?

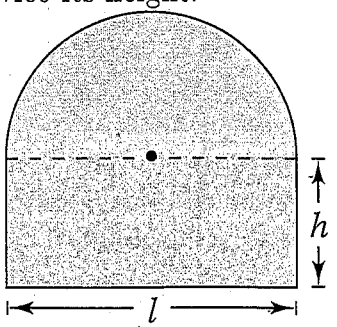
$$1 + \frac{1}{2}\cdot\frac{19}{7} + \frac{2!}{3^2}\left(\frac{19}{7}\right)^2 + \frac{3!}{4^3}\left(\frac{19}{7}\right)^3 + \frac{4!}{5^4}\left(\frac{19}{7}\right)^4 + \cdots$$

7. Find the Maclaurin series for

$$f(x) = \ln\frac{1+x}{1-x}$$

and determine its radius of convergence. Use the first four nonzero terms of the series to approximate $\ln 3$.

8. A semicircle is on top of a rectangle (see figure). If the area is fixed and the perimeter is a minimum, or if the perimeter is fixed and the area is a maximum, verify that the length of the rectangle is twice its height.



9. Find the volume of the solid bounded by the graphs of $z = \ln(x^2 + y^2)$, $z = 0$, $x^2 + y^2 \geq 1$ and $x^2 + y^2 \leq 4$.

10. Find the volume of the region of points (x, y, z) such that

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

～全卷完～

國立中山大學 100 學年度學士班轉學生招生考試試題

科目：普通物理【電機工程學系二年級】

Some useful constants:

Gas constant $R = 8.314$ J/mol·K	**Electron mass $m_e = 9.109\times10^{-31}$ kg**
Electric constant (permittivity) $\varepsilon_0 = 8.854\times10^{-12}$ C^2/N·m^2	**Electron charge $e = 1.602\times10^{-19}$ C**
Magnetic constant (permeability) $\mu_0 = 4\times10^{-7}$ T·m/A	**Mass of the Sun $M_S = 1.99\times10^{31}$ kg**
Gravitational constant $G = 6.674\times10^{-11}$ N·m^2/kg^2	**Radius of Earth 6.4×10^6 m**

Part I: Single Choice (25 Questions, 3% for each)

1. A block of mass m is placed on top of a light vertical spring of force constant k and pushed downward, so that the spring is compressed by x. After the block is released from rest it travels upward and then leaves the spring. To what maximum height above the point of release does it rise? (A) $kx^2/(mg)$ (B) $2kx^2/(mg)$ (C) $kx^2/(2mg)$ (D) $kx^2/(\sqrt{2}\,mg)$ (E) $\sqrt{2}\,kx^2/(mg)$

2. An object of mass m is initially at rest on a frictionless horizontal plane. If a constant power P acts on this object for t duration, how far will it travel during this interval?
(A) $\sqrt{\frac{8Pt^3}{9m}}$ (B) $\sqrt{\frac{Pt^3}{8m}}$ (C) $\sqrt{\frac{Pt^3}{9m}}$ (D) $\sqrt{\frac{8Pt^3}{m}}$ (E) $\sqrt{\frac{5Pt^3}{2m}}$

3. A satellite moves in a circular orbit just above the surface of a planet, assumed to offer no air resistance. If the satellite's orbit speed is v, the escape velocity from the planet is
(A) v (B) $\sqrt{2}\,v$ (C) $\sqrt{3}\,v$ (D) $2v$ (E) $2\sqrt{2}\,v$

4. The free-fall acceleration on the surface of the Moon is about one-sixth that on the surface of the Earth. If the radius of the Moon is about $0.250R_E$, the ratio of their average densities, $\rho_{\text{Moon}}/\rho_{\text{Earth}}$, is (A) 2/3 (B) 1/2 (C) 1/3 (D) 1/6 (E) 1/4

5. A solid wheel with mass M, radius R, and rotational inertia $MR^2/2$, rolls without sliding on a horizontal surface. A horizontal force F is applied to the axle and the center of mass has an acceleration a. The magnitudes of the applied force F and the frictional force f of the surface, respectively, are:
(A) F = Ma, f = 0 (B) F = Ma, f = Ma/2 (C) F = 2Ma, f = Ma
(D) F = 2Ma, f = Ma/2 (E) F = 3Ma/2, f = Ma/2.

6. A spherical shell has inner radius R_1, outer radius R_2, and mass M, distributed uniformly throughout the shell. The magnitude of the gravitational force exerted on the shell by a point particle of mass m located a distance d from the center, outside the inner radius and inside the outer radius, is:
(A) 0 (B) GMm/d^2 (C) $GMm/(R_2^3-R_1^3)$
(D) $GMm(d^3-R_1^3)/d^2(R_2^3-R_1^3)$ (E) $GMm/(d^3-R_1^3)$

7. A satellite of mass m circles a planet of mass M and radius R in an orbit at a height $2R$ above the surface of the planet. What minimum energy is required to change the orbit to one for which the height of the satellite is $3R$ above the surface of the planet?
(A) $\frac{GmM}{24R}$ (B) $\frac{GmM}{15R}$ (C) $\frac{GmM}{12R}$ (D) $\frac{2GmM}{21R}$ (E) $\frac{3GmM}{5R}$

背面有題

8. A simple pendulum is suspended from the ceiling of an elevator. The elevator is accelerating upwards with acceleration a. The period of this pendulum, in terms of its length L, g, and a is:
(A) $2\pi\sqrt{(L/g)}$ (B) $2\pi\sqrt{(L/(g+a))}$ (C) $2\pi\sqrt{(L/(g-a))}$
(D) $2\pi\sqrt{(L/a)}$ (E) $(1/2\pi)\sqrt{(g/L)}$.

9. A point source emits sound waves with a power output of 100 watts. What is the sound level (in dB) at a distance of 10 m? (A)139 (B)119 (C)129 (D)109 (E)10

10. Two harmonic waves traveling in opposite directions interfere to produce a standing wave described by $y = 2 \sin(\pi x) \cos(3\pi t)$ where x is in m and t is in s. What is the distance (in m) between the first two antinodes? (A)8 (B)2 (C)4 (D)1 (E)0.5

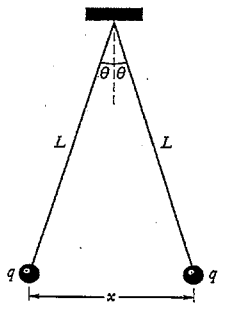
11. Work done in an isobaric process by an ideal gas is
(A) $nRT_2 \ln(V_2/V_1)$ (B) $nRT_2(1-V_1/V_2)$ (C) $nRT_2 \ln(V_1/V_2)$ (D) $nRT_1(1-V_2/V_1)$
(E) $nRT_1(1-V_1/V_2)$

12. An ideal gas is allowed to expand adiabatically. Assume the process is reversible. The change in entropy is:
(A) 0 (B) $nR \ln(V_2/V_1)$ (C) $nR \ln(T_2/T_1)$ (D) $kn \ln(V_2/V_1)$ (E) $kn \ln(T_2/T_1)$

13. A Carnot refrigerator extracts 35 kJ as heat during each cycle, operating with a coefficient of performance of 4.60. What is the energy per cycle transferred as heat to the room?
(A) 15.0 kJ (B) 21.0 kJ (C) 25.9 kJ (D) 30.4 kJ (E) 42.6 kJ

14. Three particles, each with positive charge Q, form an equilateral triangle, with each side of length d. Then the magnitude of the electric field produced by the particles at the midpoint of any side is
(A) $Q/(4\pi\varepsilon_0 d^2)$ (B) $Q/(3\pi\varepsilon_0 d^2)$ (C) $2Q/(3\pi\varepsilon_0 d^2)$
(D) $Q/(\pi\varepsilon_0 d^2)$ (E) $4Q/(3\pi\varepsilon_0 d^2)$.

15. Two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L. Assume that θ is small, then the equilibrium separation x is approximately
(A) $q^2L/(4\pi\varepsilon_0 mg)$ (B) $[q^2L/(4\pi\varepsilon_0 mg)]^{1/2}$ (C) $2[q^2L/(\pi\varepsilon_0 mg)]^{1/2}$
(D) $[q^2L/(2\pi\varepsilon_0 mg)]^{1/3}$ (E) $3[q^2L/(\pi\varepsilon_0 mg)]^{1/3}$



16. Two conducting spheres have radii R_1 and R_2, with R_1 greater than R_2. If they are far apart the capacitance is proportional to: (A) $R_1 R_2/(R_1-R_2)$ (B) $R_1^2-R_2^2$ (C) $(R_1-R_2)/R_1 R_2$
(D) $R_1^2+R_2^2$ (E) none of these

17. A conductor of radius r, length and resistivity has resistance R. It is melted down and formed into a new conductor, also cylindrical, with one fourth the length of the original conductor. The resistance of the new conductor is
(A) $\frac{1}{16}R$ (B) $\frac{1}{4}R$ (C) R (D) $4R$ (E) $16R$

18. Faraday's law states that an induced emf is proportional to:
(A) the rate of change of the magnetic field. (B) the rate of change of the electric field
(C) the rate of change of the magnetic flux (D) the rate of change of the electric flux
(E) zero

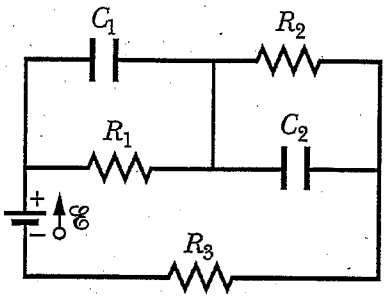
19. If an electron travels with speed **v** around a circle of radius **r**, then the magnitude of the orbital magnetic dipole moment is : (A) evr/2 (B) ev/r (C) ev/2πr (D) 2πer/v (E) 2πev/r

20. An LC circuit consists of a 1-μF capacitor and a 4 mH inductor. Its oscillation frequency is approximately: (A) 0.025 Hz (B) 25 Hz (C) 60 Hz (D) 2500 Hz (E) 15800 Hz

21. An electric field of approximately 100 V/m is often observed near the surface of Earth. If this were the field over the entire surface, what would be the electric potential of a point on the surface of Earth? (set V=0 at infinity)
(A)0 (B) 1.6×10^{8} V (C) 3.2×10^{8} V (D) 6.4×10^{8} V (E) 1.28×10^{9} V

22. A 2 kW heater element from a dryer has a length of 80 cm. If a 10 cm section is removed, what power is used by the now shortened element at 120 V?
(A) 1.75 kW (B) 2.0 kW (C) 2.3 kW (D) 2.8 KW (E) 3.2 kW

23. In the circuit, R_1=5 Ω, R_2=10 Ω, R_3=15 Ω, C_1=5 μF, C_2=10 μF and the ideal battery has *emf* =20 V. Assuming that the circuit is in the steady state, then the total energy stored in the two capacitors is
(A) 2.78×10^{-5} J (B) 1.12×10^{-4} J (C) 2.22×10^{-4} J
(D) 2.50×10^{-4} J (E) 4.72×10^{-4} J



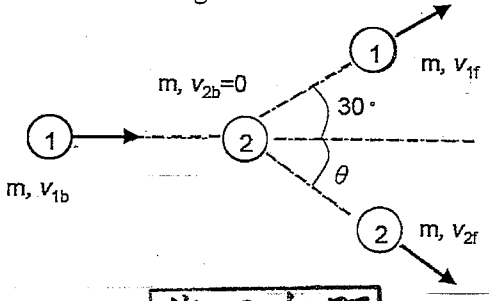
24. An electron has velocity $\boldsymbol{v}$ = (32 **i**+40 **j**)km/s as it enters a uniform magnetic field $\boldsymbol{B}$ = (60 **i**)μT.
(A) The radius of the helical path taken by the electron is 0.38 m.
(B) The pitch of that path is 1.9 m.
(C) The period of the revolution is 60 μs.
(D) To an observer looking into the magnetic field region from the entrance point of the electron, the electron spirals counterclockwise as it moves.
(E) The magnetic force points in the +**k** direction.

25. What is the maximum radiation pressure exerted by sunlight in space (S = 1350 W/m^2) on a highly polished silver surface?
(A) 1.4×10^{-2} Pa (B) 0.12 Pa (C) 9.0×10^{-6} Pa (D) 4.5×10^{-5} Pa (E) 2.3×10^{-6} Pa

Part II: Multiple Choice (5 Questions, 5% for each)

26. Consider the problem of elastic collision as shown in Fig.26. The masses of the two pucks are equal. Before colliding, the velocity of puck-1 is v_{1b}, and puck-2 is at rest initially (v_{2b}=0). Which answers are correct in the followings?
(A) $v_{2f}\cos\theta+\sqrt{3}\,v_{1f}/2=v_{1b}$ (B) $v_{2f}\sin\theta+v_{1f}/2=v_{1b}$
(C) $v_{2f}\cos\theta=v_{1f}/2$ (D) θ =60°
(E) θ = 30°

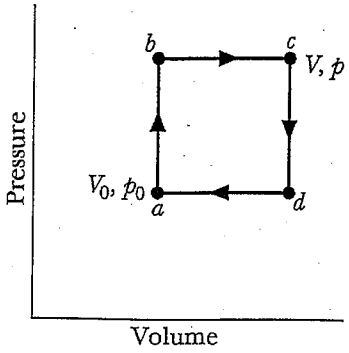
Fig. 26



背面有題

27. Figure 27 shows a reversible cycle through which 1 mol of a monatomic ideal gas is taken. Assume $p=2p_0$, $V=2V_0$, $p_0=1.01\times10^5$ Pa, and $V_0=0.0225$ m^3.

Fig. 27



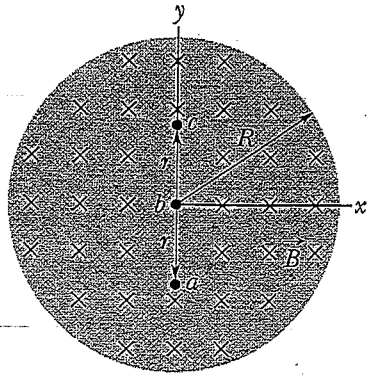
(A) The work done during the cycle is 2.27 kJ.
(B) The energy added as heat during stroke *abc* is 14.8 kJ.
(C) The efficiency of the cycle is 0.217.
(D) The efficiency of a Carnot engine operating between the highest and lowest temperatures that occur in the cycle is 0.25.
(E)The efficiency of Carnot engine is thus greater than the efficiency in the cycle showed in Fig. 27.

28. A solid nonconducting sphere of radius $R = 5.6$ cm has a nonuniform charge distribution of volume charge density $\rho = (14.1\ \text{pC/m}^3)r/R$, where r is radial distance from the sphere's center.
(A) The shell theorem cannot be applied in this case.
(B) The sphere's total charge is 7.78×10^{-15} C.
(C) The magnitude of the electric field at $r = 0$ is zero.
(D) The magnitude of the electric field at $r = R/2$ is 5.58×10^{-3} N/C.
(E) The magnitude of the electric field at $r = R$ is 1.12×10^{-2} N/C.

29. A capacitor of capacitance C_1=6 μF is connected in series with a capacitor of capacitance C_2=4 μF, and a potential difference of 200V is applied across the pair.
(A) The equivalent capacitance is 2.4 μF.
(B) Charge q_1 on capacitor 1 is 4.8×10^{-4} C.
(C) The potential difference V_1 on capacitor 1 is 80 V.
(D) Charge q_2 on capacitor 2 is 4.8×10^{-4} C.
(E) The potential difference V_2 on capacitor 2 is 120 V.

30. Figure 30 shows a uniform magnetic field $\boldsymbol{B}$ confined to a cylindrical volume of radius R. The magnitude of $\boldsymbol{B}$ is decreasing at a constant rate of 10 mT/s. In unit-vector notation,

Fig. 30



(A) the initial acceleration of an electron released at point a (r=5 cm) is $(4.4\times10^7\ \mathbf{i})$m/s^2.
(B) the initial acceleration of an electron released at point b (r=0) is nonzero.
(C) the initial acceleration of an electron released at point c (r=5 cm) is $(4.4\times10^7\ \mathbf{j})$m/s^2.
(D) the electric field at point a (r=5 cm) is $(+2.5\times10^{-4}\ \mathbf{j})$V/m.
(E) the electric field at point c (r=5 cm) is $(-2.5\times10^{-4}\ \mathbf{j})$V/m.