

Answer the following five questions, equally weighted

請依序作答

1.(20%)

Suppose that the n variables X_1, \dots, X_n form a random sample from a uniform distribution on the interval $(0, 1)$. Let the random variables Y_1 and Y_n be defined as follows:

$$Y_1 = \min\{X_1, \dots, X_n\}, \text{ and } Y_n = \max\{X_1, \dots, X_n\}.$$

The random variable $W = Y_n - Y_1$ is called the *range* of the sample. Please determine the *p.d.f.* (probability density function) of W .

2.(20%)

Suppose that X has a continuous distribution for which the *p.d.f.* is as follows:

$$f(x) = \begin{cases} 4x^3 & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the median of this distribution.

3.(20%)

Let $\mathbf{z} = (X, Y)'$ be an 2×1 random vector with

$$E(\mathbf{z}) = E \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix} = \boldsymbol{\mu},$$

and

$$E \left[\begin{pmatrix} X - \mu_X \\ Y - \mu_Y \end{pmatrix} \begin{pmatrix} X - \mu_X \\ Y - \mu_Y \end{pmatrix}' \right] = \begin{bmatrix} \sigma_X^2 & \sigma_{X,Y} \\ \sigma_{Y,X}^2 & \sigma_Y^2 \end{bmatrix} = \boldsymbol{\Sigma}.$$

If the joint density of \mathbf{z} is in the form of

$$f(\mathbf{z}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = (2\pi)^{-n/2} |\boldsymbol{\Sigma}|^{-1/2} \exp(-1/2)(\mathbf{z} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{z} - \boldsymbol{\mu}),$$

then we say that \mathbf{z} follows a bivariate normal distribution, denoted as $\mathbf{z} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Suppose that the means of X and Y are unknown but the variances of X and Y and the covariance between X and Y are known. Find the *M.L.E.'s*

(maximum likelihood estimators) of the means from a random sample of size N .

4.(20%)

Suppose that the five random variables X_1, \dots, X_5 are i.i.d. and that each has a standard normal distribution. Determine a constant c such that the random variable

$$\frac{c(X_1 + X_2)}{(X_3^2 + X_4^2 + X_5^2)^{1/2}}$$

will have a t distribution.

5.(20%)

Let X_1, X_2, \dots, X_n and Y_1, Y_2, \dots, Y_m be independent random sample from *p.d.f.'s* $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively. Assume that σ_X^2 and σ_Y^2 are known. Derive an expression for a $100(1 - \alpha)\%$ confidence interval for $\mu_X - \mu_Y$.

1. (10 分) Consider the strategic form game below

		Player 2	
		L	R
Player 1	U	3, 1	2, 4
	D	4, 2	0, 0

Find out the three equilibria of the game.

2. (10 分) Two firms are involved in developing a new technology that will allow consumers to taste food over the Internet. This has potential, for example, in restaurant promotion. Given the risks and the relatively small expected size of this market, compatibility of the technologies is very important. Firm DigiTaste is far advanced in developing its RemoteTaste technology. WebOdor has been expanding into the Internet taste arena with its incompatible product, BitterWeb. The two companies agree that if they both adopt the same technology, they each may gross \$200M from the developing industry. If they adopt different technologies, consumers will make fun of both companies, and purchase neither product, leading to a gross of \$0. Retooling one's factory to make the competing (nonproprietary) technology would cost WebOdor \$100M and DigiTaste \$250M. By the wave of an economist's wand, their production decisions must be made simultaneously.
- Set up the above scenario as a normal form (simultaneous) game. (4 分)
 - What is the equilibrium outcome? (3 分)
 - What is the implication of having higher switching costs? (3 分)
3. (10 分)
- What is the Coase theorem? (5 分)
 - Discuss its implications. (5 分)

4. (10分) Consider the following model of a lake where all members of a community have the right to fish. Assumptions and definitions are given as follows.

$$1) \quad q = f(L) = f(\sum L_i), f' > 0, \forall L < \hat{L}, f'' < 0, \hat{L} \equiv \arg \max_L f(L);$$

$$2) \quad q_i = f(L) \cdot \frac{L_i}{L}; \text{ and}$$

$$3) \quad \pi_i = L_i(p \frac{q}{L} - w), \pi_i > 0 \text{ for } L_i > 0, \text{ if and only if } (p \frac{q}{L} - w) > 0.$$

- a) Write down the condition of an individual's optimal decision; (5分)
 b) Discuss the argument that free access always leads to overfishing if there are diminishing returns. (5分)

5. (10分) Consider a two-period portfolio problem characterized by the following model.

$$\text{Maximize } EU(A R + W),$$

where W is the initial wealth, A is amount of wealth invested in the risky asset, i.e. $W-A$ is invested in the riskless asset. The rate of return for the risky asset is R , which is a random variable. The rate of return for the riskless asset is 0.

There is a mean-preserving spread in R with the following property:

$$R \rightarrow R + h(R - \bar{R}), \text{ and } h > 0.$$

Discuss the impact of such a change on the investment in the risky asset for a typical person with decreasing absolute risk aversion.

6. (15 pts) A quadratic profit function $\pi(Q) = hQ^2 + jQ + k$ to be used to reflect the following assumptions:
- (a) (5 pts) If nothing is produced, the profit will be negative;
 - (b) (5 pts) The profit function is strictly concave;
 - (c) (5 pts) The maximum profit occurs at a positive output level.
- What parameter restrictions are called for?
7. (20 pts) Recently, "How should we fix the public-provided health insurance system?" is a hot topic. If the consumers in an economy only spend on medical services (X) and a commodity (Y), please discuss the following situations by graphs. (Commodity Y is a good that if you do not consume enough of it, certain death is guaranteed.)
- (a) (10 pts) If the subsidies on the medical services are financed by a lump-sum income tax rate (α) and everyone has the same income level and preference, does a consumer receive a higher or lower utility level after the public-provided health insurance system is implemented?
 - (b) (10 pts) Now, suppose we can separate people into two income levels, Rich and Poor. Assume that we have equal number of people in these two groups. The government decides to give a higher lump-sum tax rate (β) to the Rich and a lower lump-sum tax rate (γ) to the Poor. If the Rich treats the medical services as necessities and the Poor treats them as luxuries, what is the relationship among α, β and γ given that the subsidies on the medical services are still fully financed by the tax revenues?
8. (15 pts) A monopolist faces a competitive fringe at P_0 , where $P_0 > P_c$. (By definition, a competitive fringe is willing to supply any demand at price P_0 but supplies nothing for a lower price.) P_c is the competitive price. Please use a graph to illustrate the demand curve and the optimal two-part tariff.

國立中山大學99學年度碩士班招生考試試題

科目：總體經濟學【經濟所碩士班】

一、請依題意選出正確答案 (50%，每小題各 5 分，答錯不倒扣分數。)

請注意：第 1 - 7 小題為單一選擇；第 8 - 10 小題為多重選擇。

1. If the labor force participation rate is rising, a) the size of the labor force is rising. b) the number of unemployment people is rising and the size of the labor force is falling. c) the size of the labor force is falling and the size of the working-age population is rising d) the size of the number of unemployed people is falling and the size of the labor force is rising.
2. If the real interest rate is positive, lenders a) are not compensated for the loss of purchasing power of their funds caused by inflation. b) are made worse off by lending because they can buy fewer goods when the loan is repaid than they could before they loaned funds. c) will be unwilling to lend because a loan will make them worse off. d) are compensated beyond what they must earn to compensate them for inflation.
3. Technological progress will a) shift the long-run aggregate supply (LAS) curve to the right but will not shift the short-run aggregate supply (SAS) curve. b) not shift either the LAS or the SAS curve. c) shift both the LAS and SAS curves to the right. d) shift the SAS curves to the right but will not shift the LAS curve.
4. In the circular flow diagram, if the services of labor flow clockwise, then wages flow a) clockwise. Spending on goods flows clockwise. b) clockwise. Spending on goods flows counterclockwise. c) counterclockwise. Spending on goods flows clockwise. d) counterclockwise. Spending on goods flows counterclockwise.
5. A cut in taxes causes a(n) ___ in GDP, a(n) ___ in savings, and a(n) ___ in interest rates, which ___ the private sector surplus. a) increase; increase; increase; increase. b) increase; decrease; decrease; increase. c) increase; increase; decrease; increase. d) decrease; decrease; decrease; decrease. e) decrease; decrease; increase; decrease.
6. Because of automatic stabilizers, when income falls a) governmental expenditures fall and tax revenues rise. b) governmental expenditures rise and tax revenues fall. c) governmental expenditures equal tax revenues. d) the economy will automatically go to full employment. e) none of the above is correct.
7. If European interest rates fall, a) the supply of U.S. dollar assets shifts rightward. b) the supply of U.S. dollar assets shifts leftward. c) the demand for U.S. dollar assets shifts rightward. d) the demand for U.S. dollar assets shifts leftward. e) both supply of and demand for U.S. dollar assets shift rightward. f) both supply of and demand for U.S. dollar assets shift leftward.

請注意：第 8 - 10 小題為多重選擇

8. A money market mutual fund a) is essentially the same as a demand deposit account. b) equals to a time deposit of a large amount. c) is a financial institution that issues shares redeemable by check. d) invests in residential mortgages. e) invests in long-term government securities. f) invests in highly liquid assets.
9. The rational expectations theory holds that a) the forecasts people make are accurate. b) the forecasts people make are the same as the forecasts made by an economist using the relevant economic theory. c) forecasts of the inflation will never be wrong if all available information is used. d) the expected absolute value of the forecast error is zero. e) the forecasts people make are correct on average. f) A current expectation of the CPI in 2010 is rational only if it will not change during the year.
10. In monetarist business cycle theory, a) the aggregate supply curve is horizontal. b) the aggregate supply curve has a positive slope. c) the aggregate supply curve is vertical. d) the impulse for a business cycle is changes in the money supply. e) money wages are assumed to be temporarily sticky. f) increases in the money growth rate cause real GDP temporarily to rise because real and nominal interest rates fall.

二 (25%) Please answer the following questions using Solow growth model

(6%) A. $Y = F(K, AL) = K^\alpha (AL)^{1-\alpha}$, $0 < \alpha < 1$

The Cobb-Douglas production function contains physical capital (K), technology (A), and labor (L). The constant return scale property is hold in this case. Please prove Inada condition is hold in this case.

(6%) B. If technology and labor accumulation follows $\dot{A}(t) = gA(t)$ and $\dot{L}(t) = nL(t)$, please find growth rate of Y and Y/L.

(6%) C. The capital K represents total stock of physical capital for a country. Let's assume the capital with respect to time is $\dot{K}(t) = sY(t) - \delta K(t)$, when s and delta represents saving and depreciation rate of capital. Please derive dynamics of capital per unit effective labor.

(7%) D. What is difference between absolute convergence in Solow growth model?

三 (25%) Please apply the following system equation to answer the questions

$$E_t \sum_{j=0}^{\infty} e^{-\rho t} u(c_{t+j}, 1-l_t), \quad U_t = \ln c_t + b \ln(1-l_t)$$

The representative agents gain utility from consumption and leisure. C and l represents consumption and labor hours in the utility function. The budget constrain

for agent is $w_t l_t = c_t \dots$

Please solve the following problem

(12%) A. Please maximize utility for the agent's labor hours are unrelated to wage in period 0.

(13%) B. If the agent needs to make decisions between two periods, the budge constraint $c_1 + \frac{1}{1+r} c_2 = w_1 l_1 + \frac{1}{1+r} w_2 l_2$, when r and w represents real interest rate and wage, respectively. Please prove the agents would like to take more leisure when wage rate is rising in the second period. Please also show the Euler equation and explain why the agents delay consumption in period one. [Hint: you need to rely on Lagrangian maximization process].