

請依序且橫向作答

Answer the following five questions, equally weighted

1.(20%)

Suppose that  $P(A) = 0.6$ ,  $P(B) = 0.5$ ,  $P(C) = 0.4$ ,  $P(A \cap B) = 0.3$ ,  $P(A \cap C) = 0.2$ ,  $P(B \cap C) = 0.2$ , and  $P(A \cap B \cap C) = 0.1$ .

(a). Find  $P((A \cup B) \cap C)$  and  $P(A \cup (B \cap C))$ .

(b). Find  $P((A^c \cup B^c) \cap C^c)$  and  $P((A^c \cap B^c) \cup C^c)$ , where  $A^c$  means the complement of set  $A$ .

2.(20%)

Let  $X$  and  $Y$  be independent with moment-generating functions  $M_X(t) = e^{t^2}$  and  $M_Y(t) = e^{2t^2+t}$ .

(a). Find the mean and variance of  $X$ .

(b). Find the moment-generating function of  $X + 2Y + 3$ .

3.(20%)

Let  $X_1, X_2$ , and  $X_3$  be independent, with  $X_1 \sim N(1, \sigma^2)$ ,  $X_2 \sim N(-1, \sigma^2)$ , and  $X_3 \sim N(0, \sigma^2)$ . Find the distribution of  $q$ , where

$$q = \frac{X_1^2 + X_2^2 + 2X_3^2 + 2X_1X_2}{2}$$

4.(20%)

Suppose that  $X_1, \dots, X_m$  form a random sample from a normal distribution with mean  $\mu_1$  and variance  $\sigma^2$ ; and that  $Y_1, \dots, Y_n$  form an independent random sample from a normal distribution with mean  $\mu_2$  and variance  $2\sigma^2$ . Let  $S_X^2 = \sum_{i=1}^m (X_i - \bar{X}_m)^2$  and  $S_Y^2 = \sum_{i=1}^n (Y_i - \bar{Y}_n)^2$ . Determine the values of  $\alpha$  and  $\beta$  for which  $\alpha S_X^2 + \beta S_Y^2$  will be an unbiased estimator with minimum variance.

5.(20%)

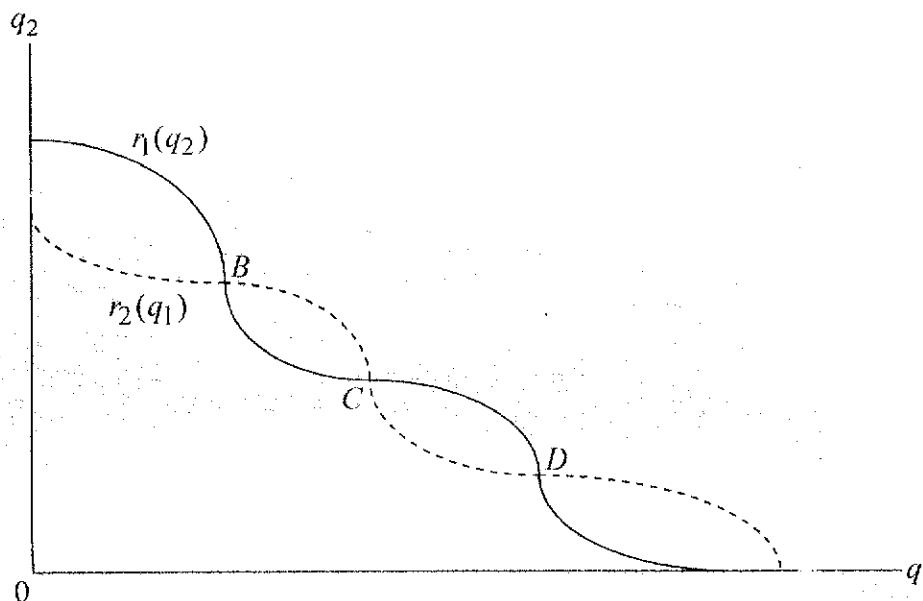
Let  $X_1, X_2, \dots$  be a sequence of discrete random variables such that  $X_n$  has density function given by  $f_n(0) = 1 - (1/n)$  and  $f_n(n) = 1/n$ . Find

(a).  $\lim_{n \rightarrow \infty} E(X_n)$  and  $\lim_{n \rightarrow \infty} Var(X_n)$ ;

(b).  $p \lim X_n$ . (i.e. the probability limit of  $X_n$ ).

Questions 1~5 are multiple selections with each scoring 10 points. For each question, only the exact answer will get the scores of 10, otherwise nil.

1. Two firms in duopoly are involved in Cournot competition. The reaction function of firm 1 is denoted as  $r_1(q_2)$ , and that of firm 2 as  $r_2(q_1)$ , as shown in the figure below. The two functions intersect at three points, B, C, and D. Which of the following statements are true?



- (a) B is a Cournot-Nash equilibrium;
  - (b) C is a Cournot-Nash equilibrium;
  - (c) D is a Cournot-Nash equilibrium;
  - (d) B and C are both asymptotically stable;
  - (e) C and D are both asymptotically stable;
  - (f) B and D are both asymptotically stable;
  - (g) C is neither a Cournot-Nash equilibrium nor asymptotically stable.
2. Consider Hotelling's model with spatial differentiation. Assume that there are two retailing stores selling the same commodity and engaging in price competition. The unit cost of each commodity is  $c$ . Consumers are evenly distributed along a street, standardized as 1 unit, but the two stores locate at both ends of the street respectively, as shown by the figure below. Each consumer has to consume one unit of the commodity. In addition to the price, a consumer has to pay the traveling costs,  $t d$ , where  $t$  is a constant and  $d$  is the distance between her location and the retail store. Pick up the true statements.

store 1

store 2

0

the street

1

- (a) The equilibrium price is  $c$ ;
  - (b) The equilibrium price is  $c + t$ ;
  - (c) The equilibrium price is  $c + t/2$ ;
  - (d) The equilibrium price is  $c + t/4$ ;
  - (e) Both stores will not charge the same price in equilibrium;
  - (f) There are multiple equilibria;
  - (g) In equilibrium, Bertrand Paradox will prevail;
  - (h) Both stores will charge the same price in equilibrium
3. Wang and Chen live in a two-person community. They have the same utility function,  $U(x, G) = xG$ , in which  $x$  is a private good and  $G$  is a public good with  $G = G_1 + G_2$ .  $G_1$  is the amount of public good contributed by Wang, and  $G_2$  is that contributed by Chen. Wang's budget constraint is:  $x + G_1 = 30$ . And Chen's budget constraint is  $x + G_2 = 50$ . Assume that  $G$  is provided voluntarily by each individual. Find the true statements below.
- (a) The amount of  $G$  Chen will contribute is 6 times of that contributed by Wang;
  - (b) Chen will consume more private goods than will Wang;
  - (c) The total amount of  $G$  being provided is  $80/3$ ;
  - (d) Wang and Chen will contribute the same amount of public goods;
  - (e) Socially efficient amount of  $G$  is 1.5 times of that provided voluntarily;
4. In the presence of negative production externalities, e.g. environmental pollution, which methods are generally suggested by economists to deal with market failure issues?
- (a) Pigouvian tax;
  - (b) Pollution-producing firms should be closed if these firms have no right to pollute by law;
  - (c) No need for the government to intervene if the property rights have been assigned clearly and transaction costs are substantial;
  - (d) Introducing a market for pollution so demanders and suppliers can trade with each other, if it is feasible;
  - (e) Quantity rationing.
5. Which statements regarding the principal-agent problems are true?
- (a) Adverse selection problems are not the key issues in these relationships;
  - (b) Moral hazard problems are the key issues in these relationships;
  - (c) Principal-agent issues have nothing to do with optimal contracts;
  - (d) Principal-agent issues are related to information asymmetry;
  - (e) There is not any principal-agent problems if there are many principals compete for one agent;
  - (f) Principals have to take into account the agent's participation constraint and incentive compatibility constraint.

6. (20 pts) You are given the following partial information about a consumer's purchases. He consumes only two goods.

|        | Year 1   |       | Year 2   |       |
|--------|----------|-------|----------|-------|
|        | Quantity | Price | Quantity | Price |
| Good 1 | 100      | 100   | 120      | 100   |
| Good 2 | 100      | 100   | Y        | 80    |

Over what range of quantities of good 2 consumed in Year 2 would you conclude:

- (a) That his behavior is inconsistent (i.e., in contradiction with the weak axiom)?

From now on, assume that the weak axiom is satisfied.

- (b) That the consumer's consumption bundle in Year 1 is revealed preferred to that in Year 2?  
 (c) That the consumer's consumption bundle in Year 2 is revealed preferred to that in Year 1?  
 (d) That there is insufficient information to justify (a), (b) and/or (c)?

7. (15 pts) Consider the utility function

$$u = 2x_1^{1/2} + 4x_2^{1/2}.$$

- (a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth.  
 (b) Find the compensated demand function.  
 (c) Find the indirect utility function, and verify Roy's identity.
8. (15 pts) An individual has Bernoulli utility function  $u(\cdot)$  and initial wealth  $w$ . Let lottery  $L$  offer a payoff of  $G$  with probability  $p$  and a payoff of  $B$  with probability  $1-p$ .
- (a) If the individual owns the lottery, what is the minimum price he would sell it for?  
 (b) If he does not own it, what is the maximum price he would be willing to pay for it?  
 (c) Are buying and selling prices equal? Find conditions on the parameters of the problem under which buying and selling prices are equal.

一、請依題意選出正確答案 (60%，每小題各 5 分，答錯不倒扣分數。)

請注意：第 1 - 9 小題為單一選擇；第 10 - 12 小題為多重選擇。

1. To qualify as money, a commodity a) must have a value in exchange greater than its intrinsic value. b) must have an intrinsic value greater than its value in exchange. c) may have a value in exchange either equal to or greater than its intrinsic value. d) must have neither intrinsic value nor value in exchange.
2. Which of the following does not cause a demand-pull inflation? a) Increases in governmental purchases. b) Increases in net exports. c) Increases in oil prices. d) Increases in the money supply.
3. The distinction between a debtor nation and a net borrower or between a creditor nation and a net lender depends on the a) distinction between the stock of borrowing or lending and current borrowing or lending. b) distinction between domestic savings and investment in other countries. c) domestic monetary base of fiat currency countries and that of commodity currency countries. d) resolution of deficits and surpluses in international accounts through the use of official and nonofficial reserves by the central bank.
4. The key to reaching high per capita income level is a) working long hours. b) attaining high rates of economic growth. c) cutting taxation rates to encourage investment. d) stimulating aggregate demand aggressively.
5. Property rights are a) the rights to use money in exchange for goods and services. b) rights that do not include the right to own financial assets. c) rights that include the right to own financial, but not physical assets. d) social arrangements that govern the ownership, use and disposal of goods and factors of production.
6. Because of an increase in expected future profit, investment increases by 500 million. In the short run, which condition increases the effects of this change on equilibrium real GDP? a) A flatter short-run aggregate supply curve. b) A steeper short-run aggregate supply curve. c) A small value for the marginal propensity to consume. d) The presence of income taxes.
7. A drop in the price level causes a) the aggregate expenditure curve to shift downward and produces movement along the aggregate demand curve. b) the aggregate expenditure to shift upward and the aggregate demand curve to shift rightward. c) the aggregate expenditure to shift upward and movement along the aggregate demand curve. d) movement along both the aggregate expenditure curve and the aggregate demand curve.

8. One feature of rational expectations theory is that a) forecasts of the inflation rate will never be wrong if all available information is used. b) the forecast error is zero. c) the expected absolute value of the forecast error is zero. d) rational expectations are correct on average.
9. Critics of the real business cycle model argue that a) the impulse for a business cycle is technological change. b) investment spending is very strongly related to the real interest rate. c) investment spending is only weakly related to the real interest rate. d) labor supply is very strongly related to the real interest rate. e) labor supply is only strongly weakly related to the real interest rate.

(請注意：第 10 - 12 小題為多重選擇)

10. Neoclassical growth theory asserts that a) technological progress is determined by investment. b) technological progress is determined by saving. c) technological progress is exogenous and a pure chance event. d) population growth does not respond to any economic forces. e) population growth rises with real wages. f) both population growth and technological progress respond to economic incentives
11. Which statement(s) of the following is (are) correct? a) The monetary base comprises currencies in circulation and commercial banks' deposits at the central bank. b) Required reserves are not a part of the monetary base. c) Gold holdings and governmental deposits at the central bank are a part of the monetary base. d) The monetary base represents the quantity of money circulating in the economy. e) The monetary base represents the system's potential for money supply expansion.
12. One reason/Reasons that the aggregate demand curve has a negative slope is/are because a) people buy more goods in the future when the price level rises. b) people buy more foreign goods when the domestic price level rises c) firms supply more when prices rise. d) nominal interest rates vary with the price level. e) nominal cash balances vary with the price level.

## 二、請回答以下問題

- 1). 解釋“Tobin's q”。(10%)
- 2). 敘述「Tobin's q 投資理論」之大意。(10%)

## 三、請根據以下模型回答下列問題

- 1). 繪圖導出總和供給曲線。(15%)
- 2). 在所繪圖形上說明，當所得稅率下降至  $t_1 < t$  時，新、舊總和供給曲線之相關位置。(5%)

總生產函數：  $Y = F(L)$ ,  $F' > 0$ ,  $F'' < 0$ . ( $F' = \frac{\partial F}{\partial L}$ ,  $F'' = \frac{\partial^2 F}{\partial L^2}$ ,  $Y =$  總產出,  $L =$  勞動)

勞動需求行爲：  $W^d = PF'(L)$ . ( $P =$  物價水準,  $W^d =$  勞動需求價格)

勞動供給行爲：  $W^s = \frac{P}{1-t}h(L)$ ,  $h' > 0$ ,  $t \in (0, 1)$ . ( $t =$  所得稅率,  $W^s =$  勞動供給價格)

勞動市場均衡條件：  $W^d = W^s$