國立中山大學經濟學研究所碩士班招生考試「總體經濟學」試題

符號說明: Y = 實質所得, C = 實質消費, I = 實質投資, R = 利率水準,
P = 物價水準, M = 貨幣數量, L = 實質貨幣需求, t = 時間,
X' = 變數X之一階導數, X_t = 第t期之X值, $X = M, Y, P, C, I, \cdots$,
$X^e_{t,t-1}$ = 在第$t-1$期時 對X_t 所形成之預測值,
ϵ_t = 第t期總合供給之 外生隨機干擾, $\epsilon_t \sim N(0, \sigma^2_\epsilon)$, σ^2_ϵ = constant.

請回答下列問題:

一、(30%)

請分析以下總體經濟模型

$$\begin{aligned} \text{I S:} \quad Y &= C(Y) + I(R), \qquad 0 < C' < 1, \quad I' < 0 \\ \text{LM:} \quad \frac{M}{P} &= L(Y, R,), \qquad L_Y = \frac{\partial L}{\partial Y} > 0, \quad L_R = \frac{\partial L}{\partial R} < 0 \end{aligned}$$

以資導出 總合需求函數 $Y = Y^d(M, P)$, 其中

$$dY = \frac{1}{\alpha}\frac{dM}{P} - \frac{1}{\alpha}\frac{M}{P^2}dP, \qquad 而 \quad \frac{1}{\alpha} = \frac{I'}{(1-C')L_R + L_Y I'}。$$

二、(20%)

考慮如下之總合供需函數

$$\begin{aligned} \text{AS:} \quad Y_t - Y_{t-1} &= \beta(P_t - P^e_{t,t-1}) + \epsilon, \quad \beta > 0 \\ \text{AD:} \quad P_t - P_{t-1} &= -\alpha(Y_t - Y_{t-1}) + M_t - M_{t-1}, \quad \alpha > 0 \end{aligned}$$

假設 $P^e_{t,t-1}$ 爲外生因素所決定之固定值, 請推算 貨幣政策乘數 $\frac{dY_t}{dM_t}$ 與 $\frac{dP_t}{dM_t}$。

三、(30%)

請考慮如上述第二題之模型, 但令 $P^e_{t,t-1}$ 爲依照「理性預期假說 (Rational Expectations Hypothesis)」所形成之價格預測值。 請推算此預測價格 $P^e_{t,t-1}$ 之形成方程式。

四、(20%)

續上述第三題, 但假設 貨幣供給函數 爲 $M_t = \bar{M}_t + \hat{M}_t$, 其中 $\bar{M}_t$與$\hat{M}_t$ 分別代表 在第$t-1$期時爲「可正確預測」與「不可正確預測」之 第t期貨幣數量的部分, 亦即:
$\bar{M}^e_{t,t-1} = \bar{M}_t$, $\hat{M}^e_{t,t-1} = \mu_t$, $\mu_t \sim N(0, \sigma^2_\mu)$, σ^2_μ = constant.
請在此貨幣供給條件下 根據「理性預期假說」推算:

1.「事先可以正確預測之貨幣政策」乘數 $\frac{dY_t}{d\bar{M}_t}$ 與 $\frac{dP_t}{d\bar{M}_t}$。
2.「事先不可正確預測之貨幣政策」乘數 $\frac{dY_t}{d\hat{M}_t}$ 與 $\frac{dP_t}{d\hat{M}_t}$。

科目：個體經濟學【經濟所碩士班】

1. Explain the following terms:
   (a) Nash equilibrium; (5%)
   (b) Transaction costs and Coase Theorem; (5%)
   (c) Pareto efficiency; (5%)
   (d) Arrow's Impossibility Theorem. (5%)

2. In a duopoly, two firms involve in Cournot competition. The cost function of each firm is given by: $c_i(q_i) = cq_i, i = 1,2.$ The market demand function is:

$$p = a - (q_1 + q_2).$$

   (a) What is the Cournot-Nash equilibrium? (5%)
   (b) What will the outcome be if the two firms form a cartel? (5%)
   (c) What will the outcome be if firm 1 acts as a Stackelberg leader? (5%)
   (d) What will the outcome be if firm 1 acts as a price leader? (5%)
   (e) What will the outcome be if the two firms are involved in Bertrand competition? (5%)

3. There are n agents with identical utility functions, $u_i(G, x_i) = G^{\alpha} x_i^{1-\alpha}$.
   Suppose that a total amount of wealth w is about to be equally divided among $k \le n$ of the agents.
   (a) How much of the public good is provided? (10%)
   (b) How does the amount of the public good change as k increases? (5%)

4. There are two players, a seller and a buyer, and two dates. At date 1, the seller chooses his investment level $I \ge 0$ at cost I. At date 2, the seller may sell one unit if a good and the seller has cost $c(I)$ of supplying it, where $c'(0) = -\infty, c' < 0, c'' > 0,$ and $c(0)$ is less than the buyer's valuation. There is no discounting, so the socially optimal level of investment, I^*, is given by $1 + c'(I^*) = 0.$
   (a) Suppose that at date 2 the buyer observes the investment I and makes a take-it-or-leave-it offer to the seller. What is this offer? (5%)
   (b) What is the perfect Nash equilibrium of the game? (10%)
   (c) Can you think of a contractual way of avoiding the inefficient outcome of (a)? (Assume that contract cannot be written on the level of I.) (5%)

5. Two consumers each with an expected utility function of $\ln w$ and $\sqrt{w}$ respectively are offered a gamble. Each consumer initially has wealth w. If one bets \$$x$, he will have $w+x$ with a probability π, and $w-x$ with a probability $(1-\pi)$. For each consumer, solve for the optimal x as a function of π. (10%)

6. Suppose that a competitive industry faces a randomly fluctuating price for its output. For simplicity we imagine that the price of output will be p_1 with probability q and p_2 with probability $(1-q)$. It has been suggested that it may be desirable to stabilize the price of output at the average price $\overline{p} = qp_1 + (1-q)p_2$. True or false? Explain why? (10%)

Answer the following four questions, equally weighted

1.(25%)
Let the 3×1 random vector $\mathbf{x}_t = (X_1, X_2, X_3)'$ follow a multivariate normal distribution,

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \sim N_3(\mu, \Sigma),$$

where

$$\mu = \begin{bmatrix} 170 \\ 68 \\ 40 \end{bmatrix} \quad and \quad \Sigma = \begin{bmatrix} 400 & 64 & 128 \\ 64 & 16 & 0 \\ 128 & 0 & 256 \end{bmatrix}.$$

Find
(a) The conditional distribution of X_1 given $X_2 = 72$, i.e. $f(X_1|X_2 = 72)$ and
(b) The conditional distribution of X_1 given $X_2 = 72$ and $X_3 = 24$, i.e. $f(X_1|X_2 = 72, X_3 = 24)$.

2. (25%)
Let X_1, X_2, X_3 be independent with X_i having density $f(x_i) = exp(-x_i), x_i > 0; \forall i = 1, 2, 3$. Let $U_1 = X_1 + X_2 + X_3$, $U_2 = X_2/U_1$, and $U_3 = X_3/U_1$. Find the joint density of U_1, U_2, U_3.

3.(25%) (This is a question of Bayesian Statistics.)
Let $X_1, \cdots, X_N$ be a sample from a normal distribution with mean Θ and variance one, and let $\Theta \sim N(a, b^2)$. Find the posterior distribution of Θ given $X_1, \cdots, X_N$.

4.(25%)
Suppose that $X_1, \cdots, X_N$ form a random sample from a uniform distribution on the interval (θ_1, θ_2), where both θ_1 and θ_2 are unknown and $-\infty < \theta_1 < \theta_2 < \infty$. Find the maximum likelihood estimators (MLE's) of θ_1 and θ_2.