Notes: Answer all the questions on the separate answer sheets provided in Chinese or English. Please label question numbers clearly, and write Legibly.

1. Assume that an economy only has two good producers: one for wheat and one for bread, and that the economy does not have a government. In a given year, the wheat producer grows 30 bushels of wheat (without using intermediate goods), of which 25 bushels are sold to the bread producer at \$3 per bushel as intermediate goods, and 5 bushels are stored as inventory to use as seed for next year's crop. The bread producer produces and sells 100 loaves of bread to consumers for \$3.5 per loaf. In addition to the bread purchased from the domestic bread producer, the consumer imports 50 loaves of bread at \$2.0 per loaf.

   a. What does GDP measure for an economy? Is GDP a stock or flow variable? [8 points]

   b. Use the expenditure approach to calculate the GDP for the year. Be specific about the quantities of C (consumption), I (investment), NX (net exports), and G (government purchases). [12 points]

2. Consider a Solow growth model. The aggregate production function, $Y = zF(K, N)$, exhibits a constant return to scale, where z is total factor productivity, K is aggregate capital, and N is the total number of workers. The production per worker is $y = \frac{Y}{N} = f(k) = zk^{\alpha}$, where $k \equiv \frac{K}{N}$ is per-worker capital and $0 < \alpha < 1$. The economy has a constant worker growth rate of n (i.e., the number of workers next period is $N' = (1 + n)N$), a constant saving rate s, and a constant capital depreciation rate of d. The law of motion for K is $K' = I + (1 - d)K$, where I is aggregate investment and K' indicates aggregate capital next period.

   a. Derive the marginal product of capital per worker (MPK). Is MPK per worker an increasing or decreasing function of capital per worker? Also, is MPK per worker an increasing or decreasing function with respect to the number of workers (N)? Draw a graph with MPK on the y-axis and k on the x-axis and explain its economic meaning. Justify your answers with mathematical derivation. [20 points]

   b. In equilibrium, aggregate investment equal aggregate saving (i.e., $I=sY$). Solve for the steady-state capital per worker as a function of exogenous parameters. Explain the economic meaning of the steady state in the context of the Solow model. [20 points]

3. Capital control has been practiced by many countries to cope with "hot money" inflows from quantitative easing adopted in several advanced economies. What are the economic rationales underlying capital control? [10 points]

4. If you were a U.K. citizen, would you vote *for* or *against* Brexit? Please defend your answer from the aspects of trade, immigration, or other macroeconomic considerations. [15 points]

5. The theme of the World Economic Forum last November is the fourth industrial revolution. What does the fourth industrial revolution refer to? What are the potential major macroeconomic impact of this revolution? [15 points]

1. (10pts) The consumer price index (CPI) is known for its upward bias. Please use a graph to explain why a worker would be over-compensated if his/her salary is adjusted by CPI each year? (For instance, the CPI is 5% and the salary is raised by 5% to compensate the worker's loss due to inflation.)

2. (10pts) A Cobb-Douglas production function is stated below:
$$q = AL^{\alpha}K^{\beta},$$
where A, α, β are all positive constants. Please show that this production function exhibits the decrease of marginal productivity on L. Can this production function exhibit increasing returns to scale? If it can, what condition do we need for that?

3. (10pts) Does profit maximization imply cost minimization? Why or why not? Does cost minimization imply profit maximization? Why or why not?

4. (10pts) OPEC last month decided to cut its oil production by 1.2 million barrels per day starting from 2017. How does this announcement affect the oil production from oil shale in the United States? Can you use a graph to explain the short-run operating decision made by the shale oil producers in the U.S.?

5. (10pts) Consider a small open economy. Suppose the world price of good A is lower than the domestic equilibrium price under autarky in this economy. Now, the government of this country decides to raise the price of good A through a tariff to protect the domestic firms that produce good A. After implementing the tariff, the domestic price of good A is still lower than the price under autarky. Please use a graph to explain the welfare change on the economy comparing to the social welfare level under free trade. If the government instead uses a quota to achieve the same domestic price level as the one under the tariff, what happen to the welfare level? Please use the same graph to explain.

6. (15pts) A child's action, $a \geq 0$, affects both her own private income, $c(a)$, and her parents' income, $p(a)$, with $c(a) < p(a)$. The child is selfish and cares only her income. The parents may transfer some income, t, to the child. The parents' utility function is $u_p = \min \ (p(a) - t, c(a) + t)$. However, the child is a rotten kid who takes an action firstly. The parents then decide how much to transfer.
    a)   (5pts) Find the equilibrium of this family game;
    b)   (5pts) What happens if $u_p = \ln(p(a) - t) + \ln(c(a) + t)$?
    c)   (5pts) Discuss the implications of the rotten kid's behavior.

7. (10pts) Consider the strategic form game in Figure 1 which shows a form of prisoner's dilemma with (D, R) being the equilibrium. Find the ranges for a and b.

<div align="center">

Player 2

		L	R
	U	4, 4	1,6
Player 1			
	D	5, 2	a,b

Figure 1

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8. (5pts) Comment on the argument that Nash equilibrium implies Pareto efficiency.

9. (20pts) Consider the following model of a lake where all members of a community have the right to fish. Assumptions and definitions are given as follows.

$$q = (\sum_{i=1}^{n} L_i)^{\alpha}, \quad q_i = q \cdot \frac{L_i}{\sum_{i=1}^{n} L_i}, \text{ and } \pi_i = p \cdot q_i - w \cdot L_i, 0 < \alpha < 1, i = 1, \ldots, n$$

where q is the total product of fish, L_i the labor input of individual i, q_i the product of individual i, p the unit price of fish, and w the competitive wage rate.

a) (5pts) Write down the condition of an individual's optimal decision;

b) (5pts) What is the condition of Pareto efficiency?

c) (5pts) Show that free access leads to overfishing; and

d) (5pts) Design a mechanism which can result in efficiency in the use of public goods, the lake.

Answer the following five questions, equally weighted

請依題序在答案卷上作答 (5 大題, 共100分)

1. (20%)　Let $f(x,y) = \frac{x+2y}{12}$, $(x,y) \in \{(0,1),(0,2),(1,0),(1,1),(2,0)\}$. Let $U = X + Y$. Find the density function of U.

2. (20%)　Let X be uniformly distributed on $(-1,1)$, and let $Y = X^2$. Find $\text{cov}(X,Y)$.

3. (20%)　Suppose that X_1, X_2, and X_3 are independent and that

$$X_1 \sim N(2,4),\ \ X_2 \sim N(3,25),\ \ X_3 \sim N(4,16).$$

Let $W = X_1 + 2X_2 - 3X_3 - 4$. Find $P(W > 0)$.

4. (20%)　Let $X_1,...,X_n$ be independent, $n \geq 2$, and $X_i \sim N(\mu,\sigma^2)$ where $\theta = (\mu,\sigma^2)$ is the unknown parameter.

   (a). Find the maximum likelihood estimator (MLE), $\hat{\sigma}^2$ for σ^2.

   (b). Is $\hat{\sigma}^2$ a consistent estimator for σ^2 ? Explain.

5. (20%)　Let $\mathbf{x} = (X_1, X_2, X_3, X_4)'$ have a four-variate normal distribution with $E(X_1) = 2$, $E(X_2) = 1$, $E(X_3) = 3$, $E(X_4) = 6$, $\text{var}(X_1)=1$, $\text{var}(X_2)=2$, $\text{var}(X_3)=3$, $\text{var}(X_4)=4$, and $\text{cov}(X_i,X_j)=1$ for all $i \neq j$.

   (a). What is the conditional distribution of $(X_1, X_2)'$, given $(X_3, X_4)'$?

   (b). What is the joint distribution of $Y_1 = X_1 + X_2 + X_3$ and $Y_2 = X_1 - 2X_4$?

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Standard Normal Probabilities
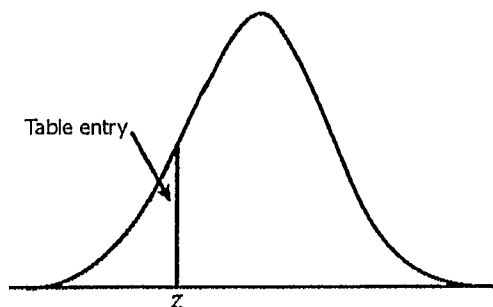
Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641