Answer the following five questions, equally weighted

請依題序在答案卷上作答 (5 大題, 共100分)

1. (20%)

    Let $\mathbf{x} = (X_1, X_2)'$ has the density function

    $$f(x_1, x_2) = \begin{cases} 10x_1 x_2^2, & 0 < x_1 < x_2,\ 0 < x_2 < 1, \\ 0, & \text{elsewhere} \end{cases}$$

    Find the density function of $W = X_1 X_2$.

2. (20%)

    In the simple linear regression model $Y = \beta_0 + \beta_1 X + u$ suppose that $E(u) \neq 0$. Letting $\alpha_0 = E(u)$, show that the model can always be written with the same slope, but a new intercept and error, where the new error has a zero expected value.

3. (20%)

    Let $Y_1, Y_2, ..., Y_n$ be a random sample from the distribution with the following probability density function:

    $$f_Y(y; \theta) = (1/2\theta^3)y^2 e^{-y/\theta}, \quad 0 < y < \infty,\ 0 < \theta < \infty.$$

    Find the MLE's of θ.

4. (20%)

    Let X and Y be two random variables with mean μ_X and μ_Y, variance σ_X^2 and σ_Y^2 and correlation coefficient ρ. Suppose that $E(X|Y) = \alpha Y + \beta$. Find α and β.

5. (20%)

    Let X and Y has a bivariate normal *pdf*,

    $$\begin{bmatrix} X \\ Y \end{bmatrix} \sim N\left(\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \right).$$

    Find the marginal distribution of X, $f_X(x)$ and the variance of the linear combination of X and Y, $(3X + 2Y)$, i.e. $Var(3X + 2Y)$.

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Notes: Answer all the questions on separate answer sheets. Label the question numbers clearly. Write Legibly.

1. Consider a Solow growth model. Let K_t be aggregate capital at the beginning of period t. Assume all the consumers in the model economy work, and the working population grows at a constant rate of $1 + n$ each period. Let the population in period t be N. Then,

$$N_{t+1} = (1 + n)N_t, \tag{1}$$

where $n > 0$. Consumers collectively receive all current real output Y_t as income, and they consume a constant fraction of income each period; aggregate consumption C_t is

$$C_t = (1 - s)Y_t, \tag{2}$$

where $0 < s < 1$ is a constant saving rate. Output is produced by a representative firm, according to the production function,

$$Y_t = zF(K_t, N_t) = zK_t^\alpha N_t^{1-\alpha}, \tag{3}$$

where z is total factor productivity (TFP) and $0 < \alpha < 1$. In this economy, investment equals saving in equilibrium. Capital depreciates each period at a constant rate $0 < \delta < 1$. The law of motion of aggregate capital is

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{4}$$

Based on the above information, answer the following questions.

   (a) Denote capital per worker by $k_t \equiv \frac{K_t}{N_t}$. What is the marginal product of capital per worker (denoted by MPK_t)? (Hint: You want to first derive the per-worker output, denoted by y_t.) [5 points]

   (b) Does MPK_t exhibit diminishing marginal product of capital? Plot MPK_t as a function of k_t, and support your answer by explicitly driving the marginal change of MPK_t with respect to k_t. [5 points]

   (c) The model's equilibrium can be boiled down to one equation that characterizes the evolution of capital per worker. Derive this equilibrium condition. [10 points]

   (d) The Solow model predicts that regardless of initial capital levels, an economy will converge to a steady state, in which capital per worker remains constant each period. Use the equilibrium condition you just derived in (c) to solve for the steady-state capital per worker, k, in terms of exogenous parameters. [10 points]

   (e) Based on your deviation of k, solve for output per worker in the steady state, y. How does the steady-state output change with respect to an increase in the saving rate? Support your answer with a graphical or an analytical approach. [10 points]

   (f) Assume that there exists another economy (Economy B), which has all aspects identical to the economy described earlier (Economy A), except that Economy B has a higher population growth rate, n' and $n' > n$. Which economy has higher steady-state consumption per worker? Support your answer by a graphical explanation or an analytical derivation. [10 points]

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2. Consider national product accounting. Let S be national savings, S^g be government savings, S^p be private-sector savings by citizens of a nation, I be aggregate investment, and CA be current account surplus. Each period, the government of the nation collects taxes (T), purchases goods (G), provides transfers to consumers (TR), and pays interest on its debt owed to the private sector (INT). The disposable income is $Y^d = Y + NFP + TR + INT - T$, where Y is GDP and NFP is net factor payments from abroad to domestic residents.

   (a) The government's receipt in this nation is T and expenditures consist of G, TR, and INT. What is government deficits (denoted by D)? [5 points]

   (b) Use the income-expenditure identity of GDP to derive the identity $S = I + CA$. (Hint: $D = -S^g$.) Also, interpret the economic meaning of $S = I + CA$. [10 points]

3. After the global financial crisis, many developed countries have entered an era of high debt and slow growth. If a government facing such a condition would like stimulate its economy, what kind of policy recommendation will you give? Any concerns in the policy prescription you just gave? [15 points]

4. In recent years, several advanced economies, such as Japan, U.S., and the euro area, have conducted quantitative easing (QE). What is the difference between QE and conventional monetary policy? [10 points]

5. The Chinese government recently abandoned its more than 35-year-old "one-child policy" for all couples. What is the economic rationale for adopting this policy 35 years ago? What is the rationale to abandon it 35 years after? [10 points]

Let $S_M = P - 20$ be the supply of imports of product X, and $S_D = \frac{3P}{2} - 30$ the supply of domestic production. S_T and $D = 100 - P$ are the total supply and demand schedules in the domestic country. Under free trade, X_F is purchased at a price P_F, with output divided between domestic production, D_F, and imports, M_F.

1. (5 pts) Write down the total supply function S_T under free trade.
2. (5 pts) What is the price P_F under free trade?
3. (5 pts) What is the amount of D_F under free trade?
4. (5 pts) What is the amount of M_F under free trade?
5. (5 pts) Measure the amount of consumers' surplus under free trade.

Now let a tariff be imposed on imports that shifts the import supply schedule to $S_M^t = \frac{5P}{6} - \frac{100}{6}$. Total supply shifts to S_T^t, and X_R is purchased at a price P_R, with output divided between domestic production, D_R, and imports, M_R.

6. (5 pts) What is the price P_R under tariffs?
7. (5 pts) What is the impact of tariffs on imports, $M_R - M_F$?
8. (5 pts) What is the change in consumers' surplus brought about by tariffs?
9. (10 pts) Draw a figure to demonstrate the welfare-loss caused by tariffs.

10. (5 pts) Explain why the social demand curve for a public good is the vertical sum of the demand curves of each individual.

11. (5 pts) Please answer True or False and explain why regarding the following statement: Firms that are most likely to buy marketable pollution rights are those that produce the most pollution per unit of output produced.

12. (10 pts) Chen is a risk-averse person and he owns a house in Kaohsiung. He wants to insure his house, which is worth NT$4 million. There is 25% of chance that his house will burn next year. If a fire occurs, the house will be worth only NT$2 million. Now, the insurance company offers Chen a policy which states that for every dollar Chen pays as the premium, he can receive two dollars in return if the house does burn. Is this policy qualified as fair insurance? If not, what amount of money should Chen receive in return for each dollar he spends as insurance premium when a fire occurs and so the policy is fair?

13. (10pts) Consider the following simultaneous-move game:

		Company B		
		Left	Middle	Right
Company A	Up	0,0	3,1	6,2
	Middle	1,3	5,5	7,4
	Down	2,6	4,7	6,6

(a) (5 pts) Is there any strictly dominated strategy for either player? Please identify it.
(b) (5 pts) Use iterated elimination of strictly dominated actions to find the unique pure strategy Nash equilibrium.

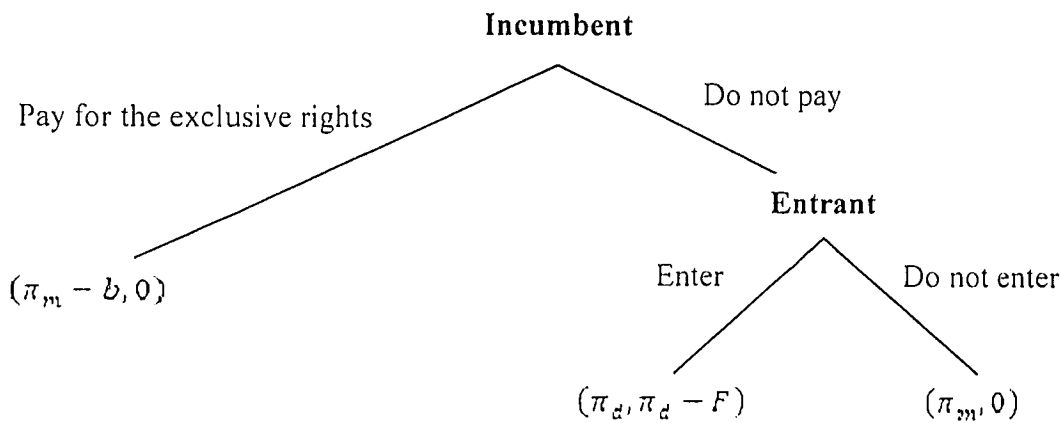14. (10pts) Consider the following extensive game with perfect information:

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Incumbent

Pay for the exclusive rights　　　　　Do not pay

$(\pi_m - b, 0)$

Entrant

Enter　　　　Do not enter

$(\pi_d, \pi_d - F)$　　　　$(\pi_m, 0)$

Please identify the subgame perfect Nash equilibrium when

(a) (5pts) $\pi_d - F \geq 0$ and $\pi_m - b < \pi_d$

(b) (5pts) $\pi_d - F \geq 0$ and $\pi_m - b \geq \pi_d$

Note: In the payoff pair, the left item represents the utility of the incumbent; the right item represents the utility of the entrant.

15. (10 pts) Suppose there are n firms that are producing the same product and each firm tries to determine the optimal production level to maximize its profit. Denote firm i's production quantity to be q_i. Assume that each firm has an identical marginal cost, c, and the market has a linear demand function, $p = a - bQ$, where $Q = \sum_i q_i$. Please calculate the equilibrium output for each firm under non-cooperative competition. Furthermore, please determine the Lerner Index and state the value of the index when n grows very large.