國立中山大學99學年度碩士班招生考試試題

科目：應用數學【物理系碩士班】

計算題(100%, 每小題 10 分)

1. (10%) The Navier-Stokes equations of hydrodynamics contain a nonlinear term $\nabla\times\left[\vec{v}\times\left(\nabla\times\vec{v}\right)\right]$, where $\vec{v}$ is the fluid velocity. For fluid flowing through a cylindrical pipe in the z-direction $\vec{v}=v(\rho)\hat{e}_z$, where $\hat{e}_z$ is the unit vector along the z-direction and $v(\rho)$ with $0\le\rho<\infty$ is a function of radial distance ρ, find $\nabla\times\left[\vec{v}\times\left(\nabla\times\vec{v}\right)\right]=?$

2. (10%) The 2×2 Pauli matrices are given by $\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}, \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ in which $i^2=-1$, and $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$. Find the matrix $U_1=\exp(\frac{ia\sigma_1}{2})$, where a is a constant.

3. (10%) Find the general solution $y(x)$ of the differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=\cos 2x.$$

4. (a) (5%) Find the Fourier series of the function

$$f(x)=x^2,\quad -\pi<x<\pi.$$

(b) (5%) Use the result of (a) to find the sum of series

$$\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n^2}\right)=?$$

5. (10%) Evaluate the following integral $\int_0^{\pi}\frac{d\theta}{1+\cos^2\theta}=?$

6. (10%) Find the Laplace transform $L\left\{J_0(t)\right\}$ of the given function

$$J_0(t)=\frac{1}{\pi}\int_0^{\pi}\cos(t\cos\theta)d\theta.$$

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科目：應用數學【物理系碩士班】

7. (10%) Please perform the saparation of variables for the following partial differential equation

$$a_1(x)\frac{\partial^2 u}{\partial x^2}+b_1(y)\frac{\partial^2 u}{\partial y^2}+a_2(x)\frac{\partial u}{\partial x}+b_2(y)\frac{\partial u}{\partial y}=0.$$

(Hint: You do not need to solve the differential equations satisfied by the independent variables.)

8. (10%) Let $f(x)=a^x$ $(a>0)$. Find its Maclaurin expansion of $f(x)$ up to order x^3.

9. (10%) Given the function of a complex variable $f(z)=z\,\mathrm{Im}\,z$, where $z=x+iy$.
   (a) (3%)Where does the derivative df/dz exists?
   (b) (3%) Where is the function analytic?
   (c) (4%) If df/dz exists at certain region, please find the derivative df/dz.

10. (10%) The linear wave equation is often written in the form

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2},$$ where v is the wave speed.

Find the general solution $y(x,t)$ of the wave equation.

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科目：電磁學【物理系碩士班】

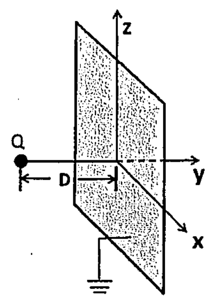
1(25%)

(a) (5%) Please <u>write down</u> the Gauss's Law in both differential and integration forms.

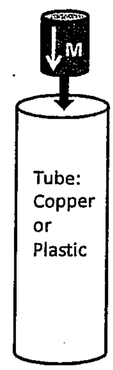
(b) (5%) Please <u>describe</u> the charge distribution and <u>draw</u> two Gauss's Spheres inside and outside of a conducting sphere with a radius of R and charge of Q.

(c) (10%) Please <u>explain in detail</u> how you transform the Gauss's integration from a vector form into a scalar form based on the Gauss's spheres in (b).

(d) (5%) Please <u>calculate</u> the electric potential inside and outside of this conducting sphere.

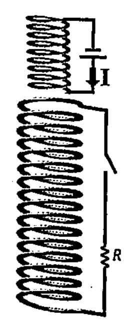


2(25%) An infinite conducting sheet is grounded as shown in the right figure. If a charge, Q, stays on the left side of this conductor sheet, D. Please <u>calculate in detail</u> what are the electric field, E, the surface distribution, σ_b, and the total value, q_b, of induced charges.



3(25%)

(a) (10%) A very strong cylindrical permanent magnet is polarized along the cylindrical axis and is dropped into vertical placed cylindrical tubes. If the tube is a copper tube, please <u>describe in detail</u> what may happen during the fall such as the <u>falling speed</u> of the magnet, the <u>responses</u> of the tube and the <u>interaction</u> between them. What may <u>happen</u> by repeat a similar experiment if the copper tube is replaced by a plastic tube? [You have to answer in detail and to list all necessary equations]



(b) (15%) Now, a small solenoid carrying a current I with n turns of coils, a length of l cm and radius of r, is dropped into a vertical placed large solenoid with N turns of coil, a length of L ($L>>l$) and radius of R ($R \geqq r$). Please <u>calculate</u> the speeds of the small solenoid *when* the large solenoid *is* and *is not* connected to a resistor R.

4(25%)

(a) (5%) Please <u>write down</u> the Maxwell's equations and their names.

(b) (5%) Please <u>write down</u> the Maxwell's equations in a source free space.

(c) (5%) Please <u>deduce</u> the wave equations of **E** and **H** in a source free space.

(d) (10%) When the EM wave incident into a lossy material (σ<<), please calculate the absorption coefficient? (You may assume all necessary parameters)

[$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$]

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科目：近代物理【物理系碩士班】

1. **(15%)** Select what was learned about quantization of radiation or mechanical system from ***two*** of the following experiments:
   (a) Photoelectric effect
   (b) Black body radiation spectrum
   (c) Frank-Hertz experiment
   (d) Davisson-Germer experiment
   (e) Compton scattering

2. **(15%)** Express briefly the following terms:
   (a) **(5%)** The tunnel effect
   (b) **(5%)** Spin-orbital interaction
   (c) **(5%)** The normal Zeeman effect

3. **(15%)** Express each of the following quantities in terms of $\hbar$, e, c, m=electron mass. Also give a rough estimate of numerical size for each
   (a) **(5%)** Bohr radius (in cm).
   (b) **(5%)** Compton wavelength of an electron (in cm).
   (c) **(5%)** Electron rest energy (in MeV).

4. **(15%)** For $\ell = 3$,
   (a) **(5%)** What is the maximum value of $L_x^2 = L_y^2$?
   (b) **(5%)** What is $L_x^2 = L_y^2$ for $\ell = 3$ and $m = 2$?
   (c) **(5%)** What is the minimum value of n that this state can have?

5. **(20%)** A particle of mass m is confined in a one-dimensional box in a region $0 \le x \le a$. At time $t = 0$ its normalized wave function is

$$\psi(x,t=0) = \sqrt{8/5a}\left[1+\cos(\frac{\pi x}{a})\right]\sin(\pi x/a).$$

   (a) **(10%)** Find the expectation value for the energy of this system at $t = 0$.
   (b) **(10%)** What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \le x \le a/2$) at $t = 0$?

6. **(20%)** A particle in a box in one-dimensional length a. The ground state: $\Psi(x,t) = A\cos\frac{\pi x}{a}e^{\frac{-iEt}{\hbar}}$.
   (a) **(10%)** Calculate A such that $\Psi(x)$ is properly normalized.
   (b) **(10%)** Calculate the expectation value of energy <x>, and <p^2>?