科目：應用數學【物理系碩士班】 ✓

選擇題(20%, 每題 5 分)

1. $\overline{AB^T}$ equals to
   (a) $\overline{B}^{-1}\overline{A}^{-1}$
   (b) $\overline{A}^T\overline{B}^T$
   (c) $\overline{A}^{-1}\overline{B}^{-1}$
   (d) $\overline{B}^T\overline{A}^T$

2. Which of following is **wrong**:
   (a) $\vec{\nabla}(f\cdot\overrightarrow{A})$
   (b) $\vec{\nabla}f\cdot\vec{A}$
   (c) $\vec{\nabla}f\times\vec{A}$
   (d) $\vec{\nabla}\cdot(\vec{B}\times\overrightarrow{A})$

3. Please find the Laplace transform of cos(ωt)
   (a) $\frac{\omega}{s^2+\omega^2}$
   (b) $\frac{s}{s^2+\omega^2}$
   (c) $\frac{s}{s^2-\omega^2}$
   (d) $\frac{\omega}{s^2-\omega^2}$

4. What is the best approximation for a periodic function, f(x), by trigonometric polynomials:
   (a) Taylor Series
   (b) Binominal formula
   (c) Fourier Series
   (d) Laplace transform

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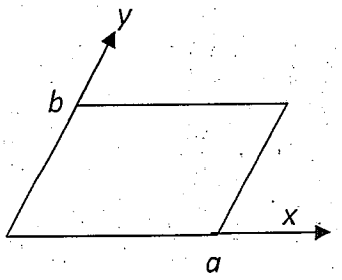
計算題(80%, 每小題 10 分)

1. (a) (10%) An ordinary differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact! If, fortunately, it can be transformed to an exact ODE by multiplying a function F(x), please find F(x).
   (b) (10%) Is the ODE $[\cos(\omega x) + \omega \sin(\omega x)]dx + e^x dy = 0$ exact?
   (c) (10%) Please solve the ODE in (b). (Please note that y(0)=1.)

2. A two dimensional membrane of lengths *a* and *b* along *x* and *y* axes. Four edges of the membrane are fixed and not allowed to vibrate. If the membrane has a initial displacement *f(x,y)* and a zero velocity at all points. When the vibration equation is described by a partial differential equation (PDE) $\nabla^2 u(x,y,t) = \frac{1}{c^2}\frac{\partial^2 u(x,y,t)}{\partial t^2}$



   where *c* is a constant and *u(x,y,t)* is the displacement of each point of the membrane.
   (a) (10%) Please use the method of separation of variables to decompose the PDE into Ordinary Differential Equations (ODEs).
   (b) (10%) Please write down all useful boundary and initial conditions.
   (c) (10%) By using the boundary conditions, please find the solutions of these ODEs?

3. (a) (10%) f(x) consists of how many poles? What are they?

$$f(x) = \frac{1}{(x^2 - 3x + 2)(x^2 + 1)}$$

   (b) (10%) Please find the principle value of the integration of f(x)

$$\text{pr.v.} \int_{-\infty}^{\infty} \frac{dx}{(x^2 - 3x + 2)(x^2 + 1)}$$

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Note: Bold characters represent vectors （粗體字代表向量）.

第一題~第六題為單選題(每題十分)；第七題~第九題為計算問答題

（請詳細作答）

1. The correct form of Ampere's law for circuits with gaps in them is

(a) $\oint \mathbf{B}\cdot d\mathbf{s} = 0$.

(b) $\oint \mathbf{B}\cdot d\mathbf{s} = I_{enclosed}$.

(c) $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{enclosed}$.

(d) $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$.

(e) $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} - \frac{\varepsilon_0}{(\mu_0)^2}\frac{d\Phi_B}{dt}$.

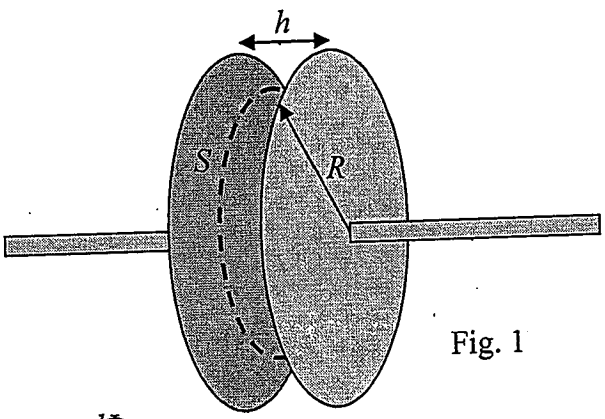

Fig. 1

2. Gauss's Law states that the net electric flux, $\oint \mathbf{E}\cdot d\mathbf{A}$, through any closed surface is proportional to the charge enclosed: $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\varepsilon_0}$. The analogous formula for magnetic fields is:

(a) $\oint \mathbf{B}\cdot d\mathbf{A} = 0$.

(b) $\oint \mathbf{B}\cdot d\mathbf{A} = \frac{q_{mag}}{\varepsilon_0}$.

(c) $\oint \mathbf{B}\cdot d\mathbf{A} = \frac{I}{\mu_0}$.

(d) $\oint \mathbf{B}\cdot d\mathbf{A} = \frac{I}{\mu_0\varepsilon_0}$.

(e) $\oint \mathbf{B}\cdot d\mathbf{A} = -\frac{d\Phi}{dt}$.

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3. In an electromagnetic wave, 1) how are the electric and magnetic field directions related and 2) how is the direction of travel determined from their directions? (**c** is the velocity of the light wave.)

(a) $\mathbf{E} \parallel \mathbf{B}$; $\frac{\mathbf{c}}{c} = \frac{\mathbf{E}\times\mathbf{B}}{|\mathbf{E}\times\mathbf{B}|}$.

(b) $\mathbf{E} \parallel \mathbf{B}$; $\frac{\mathbf{c}}{c} = \frac{\mathbf{B}\times\mathbf{E}}{|\mathbf{B}\times\mathbf{E}|}$.

(c) $\mathbf{E} \perp \mathbf{B}$; $\frac{\mathbf{c}}{c} = \frac{\mathbf{E}\times\mathbf{B}}{|\mathbf{E}\times\mathbf{B}|}$.

(d) $\mathbf{E} \perp \mathbf{B}$; $\frac{\mathbf{c}}{c} = \frac{\mathbf{B}\times\mathbf{E}}{|\mathbf{B}\times\mathbf{E}|}$.

(e) $\mathbf{E} = \mathbf{B}/c$; $\frac{\mathbf{c}}{c} = \frac{\mathbf{B}\times\mathbf{E}}{|\mathbf{B}\times\mathbf{E}|}$.

4. Show the electric field $\mathbf{E}(r,\theta,\phi)$ of an electric dipole $\mathbf{P} = p\hat{z}$ at a position r where is far away from the dipole center

(a) $\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^3}(\hat{r}2\cos\theta + \hat{\theta}\sin\theta)$

(b) $\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^2}(\hat{r}2\cos\theta + \hat{\theta}\sin\theta)$

(c) $\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^3}(\hat{r}\cos\theta + \hat{\theta}2\sin\theta)$

(d) $\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^2}(\hat{r}\cos\theta + \hat{\theta}2\sin\theta)$

(e) $\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^2}(\hat{r}\cos\theta + \hat{\theta}\sin\theta)$

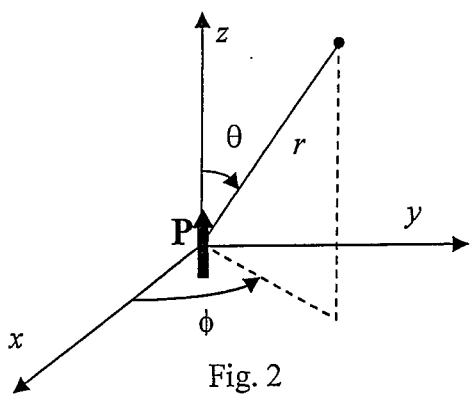

Fig. 2

5. The magnetic moment of an electron (charge = $-e$; mass = m_e) moving in a circular orbit of radius r with speed v about a nucleus of mass m_N is proportional to

(a) r/v.
(b) $m_e vr$.
(c) ev/r.
(d) evr.
(e) $m_N vr$.

6. Before the switch is closed in the figure 3, the potential across the capacitor is 200 V. At some instant after the switch is closed, the instantaneous current is 0.70 A. What is the energy in the capacitor at this instant?

(a) 49 mJ
(b) 31 mJ
(c) 80 mJ
(d) 0.13 J
(e) 62 mJ

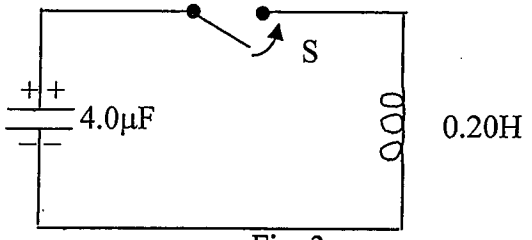

Fig. 3

7. A cylindrical capacitor of length L consists of coaxial conducting surfaces of radii a and b (Fig. 4). The dielectric material between the surfaces has a relative permittivity $\varepsilon_r = 2 + (4/r)$ for $a < r < b$. (a) Determine the capacitance of this capacitor. (b) Find the electrostatic energy stored in the dielectric region. (Neglect the fringing of the electric field at the edge.) (14%)

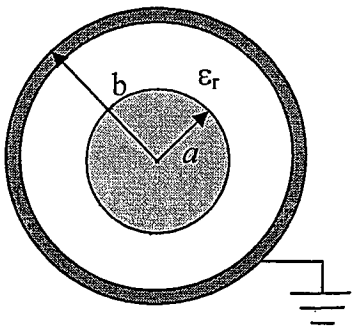

Fig. 4

8. If the constant electric field in Fig. 5 has a magnitude E_0, calculate the total electric flux through the paraboloid S. (10%)

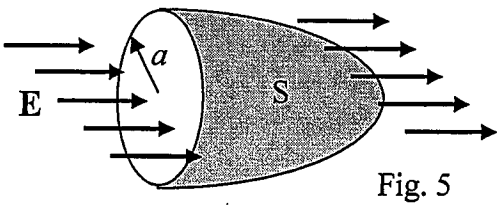

Fig. 5

9. Two ground, semi-infinite, parallel-plate electrodes are separated by a distance b. A third electrode perpendicular to and insulated from both is maintained at a constant potential V_0 (see Fig. 6). Determine the potential distribution in the region enclosed by the electrodes. (16%)

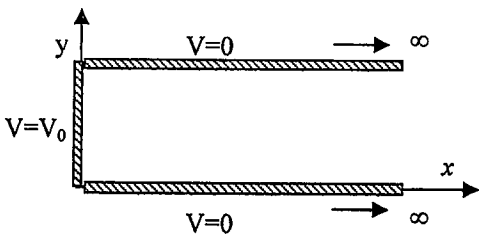

Fig. 6

科目：近代物理【物理系碩士班】 V

1. **(20%)** At time t = 0 a particle in the potential $V(X) = m\omega^2 x^2/2$ is described by the wave function

   $$\varphi(x,0) = A\sum_n (1/\sqrt{2})^n \varphi_n(x),$$

   Where $\varphi_n(x)$ are eigenstates of the energy with eigenvalues $E_n = (n+1/2)\hbar\omega$. You are given that $(\varphi_n, \varphi_{n'}) = \delta_{nn'}$.

(a) Find the normalization constant A.

(b) Write an expression for $\varphi(x,t)$ for t > 0.

(c) Find the expectation value of the energy at $t = 0$.

2. **(20%)** An electron is confined in a three-dimensional infinite potential well. The sides parallel to the *x*-, *y*-, and *z*-axes are of length *L* each.

(a) Write the Schrödinger equation.

(b) Write the time-independent wave function corresponding to the state of the lowest possible energy.

(c) Write an expression for the number of states, *N*, having energy less than some given *E*. Assume *N*>>1.

3. **(20%)** At the time t=0 the wave function for hydrogen atom is

   $$\varphi(r,0) = \frac{1}{\sqrt{10}}(2\varphi_{100} + \varphi_{210} + \sqrt{2}\varphi_{211} + \sqrt{3}\varphi_{21-1})$$

   where the subscripts are values of the quantum numbers *n, l, m*. Ignore spin and radiative transitions.

(a) Find the expectation value for the energy of this system.

(b) What is the probability of finding the system with $l = 1$, $m = +1$ as a function of time?

4. **(20%)** A certain state $|\varphi\rangle$ is an eigenstate of $\hat{L}^2$ and $\hat{L}_z$:

   $$\hat{L}^2|\varphi\rangle = l(l+1)\hbar^2|\varphi\rangle, \quad \hat{L}_z|\varphi\rangle = m\hbar|\varphi\rangle.$$

   Find the $\langle \hat{L}_x \rangle$ and $\langle \hat{L}_x^2 \rangle$ values.

5. **(20%)** A hydrogen atom is in a $^2P_{1/2}$ state with total angular momentum up along the *z*-axis. In all parts of this problem show your computations and reasoning carefully.

(a) What is the probability of finding the electron with spin down?

(b) When applying a weak magnetic field along the positive *z*-axis. What is the effective magnetic moment of the atom in this field?