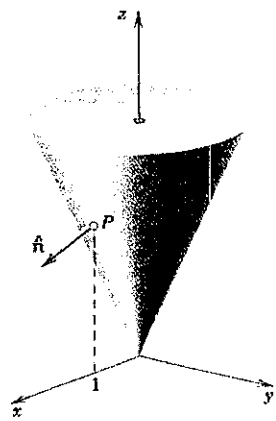
Show the details of your work.

1. (5 points) Given a cone of revolution $z^2 = 9(x^2 + y^2)$ as shown below, find a unit normal vector $\hat{n}$ at point P(1,0,3).



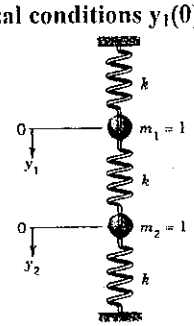
2. (5 points) Use Gauss-Jordan matrix inversion method to find the inverse matrix of

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 2 \\ 2 & 1 & 3 \end{pmatrix}$$

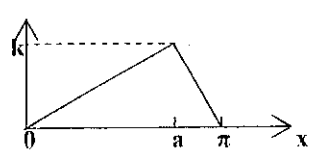
3. (10 points) Find the eigenvalues and the corresponding orthonormal eigenvectors of the matrix

$$\begin{pmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}.$$

4. (12 points) If a wet sheet in a dryer loses its moisture at a rate proportional to its moisture content, and if it loses half of its moisture during the first 10 minutes, when will it have lost 90% of its moisture?
5. (12 points) Applying convolution of Laplace transform, find the solution of $y'' + y = 3\cos 2t$; $y(0) = 0$, $y'(0) = 0$.
6. (16 points) Use Laplace transform to solve the displacements from their positions of static equilibrium $y_1(t)$ and $y_2(t)$ of two bodies of mass 1 with initial conditions $y_1(0) = 1$, $y_2(0) = 1$, $y_1'(0) = \sqrt{3k}$, $y_2'(0) = -\sqrt{3k}$.



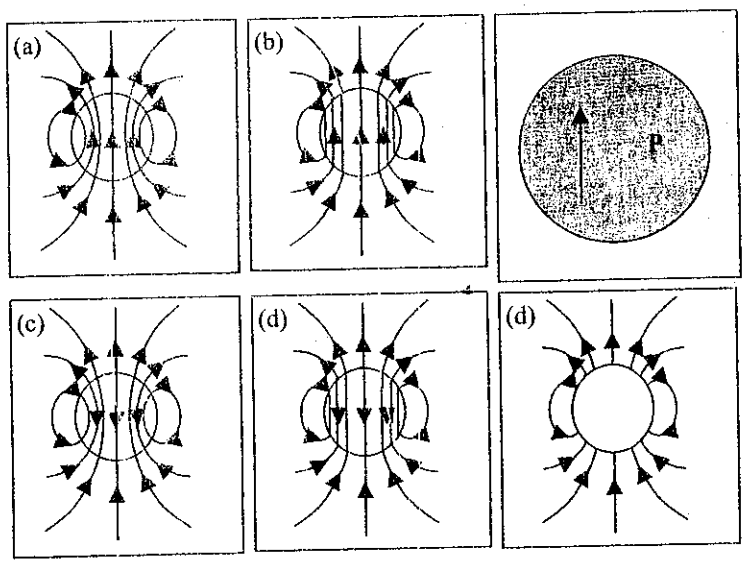
7. (15 points) Find the deflection u(x, t) of the string of length $L = \pi$ fixed at both ends. The initial velocity is zero and the initial deflection is



8. (15 points) Use Fourier transform to find the temperature u(x, t) in the infinite bar if the initial temperature is $f(x) = U_0 = \text{constant}$, $|x| < 1$, and 0, $|x| > 1$.
9. (10 points) Find the Cauchy principal value of the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2 - ix}$.

試題共分二部份：第一部份選擇題共六題，每題十分

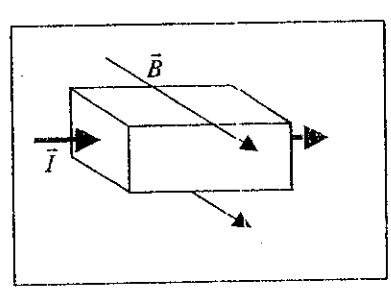
1. 一圓形介電材料，內含均勻電偶極矩($\vec{P}$)，請問圓球內外之電場分佈為下列何者？



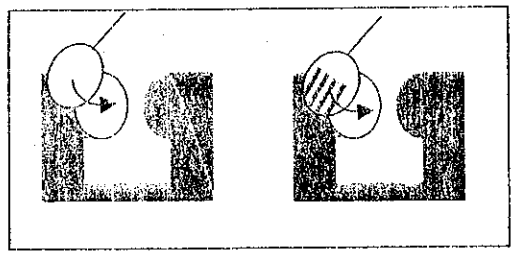
2. 在一真空腔內有一載子槍，槍口射出平行前進之載子束，以高速前進。請問，此時若無其他外力之影響下，該載子束會
   (a) 維持出槍口時之平行，繼續往前進。
   (b) 會因行進時產生之磁場而產生聚焦的現象。
   (c) 會因電荷間的庫侖排斥力遠大於磁性之勞侖茲力而產生發散現象。
   (d) 會先行因庫侖力而發散，後由勞侖茲力產生聚焦。
   (e) 會先行因勞侖茲力產生聚焦，最後與庫侖排斥力平衡，形成較小直徑之電子束，平行前進。

3. 靜磁學中之$\vec{\nabla}\cdot\vec{B}=0$代表無單磁極的存在。若在未來的某日，發現單磁極的存在，而且該式可以改寫為$\vec{\nabla}\cdot\vec{B}=\mu_0\rho_B$時，請問，物體在靜電及靜磁場中的受力方程式為：
   $\vec{F}=$
   (a) $Q_e\vec{E}+Q_e(\vec{v}\times\vec{B})$
   (b) $Q_e\vec{E}$
   (c) $Q_e\vec{E}+Q_B\vec{B}+Q_e(\vec{v}\times\vec{B})$
   (d) $Q_B\vec{B}+Q_e(\vec{v}\times\vec{B})$
   (e) $Q_B\vec{B}$

4. 外加電流以橫向流經一純 n 型半導體，垂直於電流方向並且平行於底面則是外加磁場(如圖所示)，請問樣品之上表面及下表面因霍爾效應分別累積了
   (a) +, - 電荷
   (b) +, + 電荷
   (c) -, + 電荷
   (d) -, - 電荷
   (e) 無任何累積電荷。

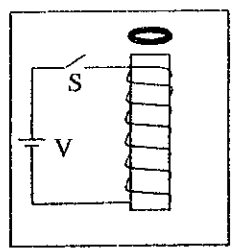


5. 二薄薄的圓鋁片以一細繩懸馬蹄型磁鐵中。其中一片以細鋸將圓片下半部切成細條狀。當輕推圓片使往回垂直於磁鐵中心連線擺盪，請問發生何事？



(a) 左邊無切口者，較快停止
(b) 右邊有切口者，較快停止
(c) 兩者停止的速度完全相同
(d) 兩者都不會停止，以相同的擺盪速度，繼續擺盪
(e) 兩者會越擺越高。

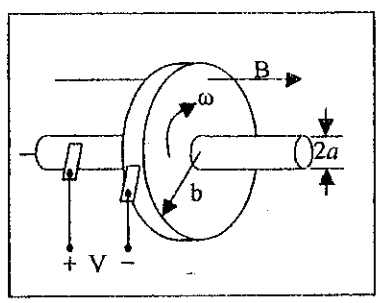
6. 將環狀線圈置於電磁鐵上。當開關 S 尚未關上時，電磁鐵無任何殘磁。當開關合上時，電磁鐵立即產生一磁場，請問此時該環狀線圈會如何反應？



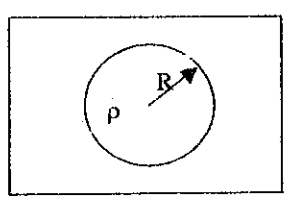
(a) 緊緊的吸在電磁鐵頂部
(b) 立即被排斥而往上跳
(c) 立即因楞次定律而原地旋轉
(d) 根本不會發生任何事
(e) 電線燒掉

第二部份計算題共三題，共 40 分

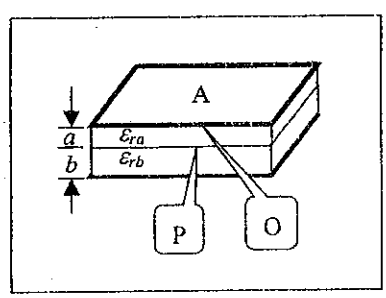
1. (10%)A Monopolar motor is assembled by conducting material made disk and cylindrical bar. Their diameters are shown in the right figure. An uniform magnetic field is supplied parallel to the cylindircal bar and perpendicular to the disk. When an external battery supplied a current I from the positive side into the bar and along the radius direction of the disk back to the negative side, please calculate the torque ($\vec{\tau}$) and the angular velocity ($\vec{\omega}$) of the disk. During the electric conduction, the electric current experiences an electric resistance of R.



2. (10%)A sphere with a radius R contains charges distributed as $\rho = ar^2 + br + c$ where ρ and r are the charge density and the distance to the center of the sphere. Please calculate the electric field ($\vec{E}$) and the electric potential (V) at r < R, r = R and r < R.



3. (20%)Two copper plates with surface area A are separated by two dielectric materials as shown in the right figure. The relative permittivities and the thickness of two dielectric materials are ε_{ra}, ε_{rb} and a, b respectively. Please find the electric displacement ($\vec{D}$), the electric field ($\vec{E}$), the polarization ($\vec{P}$), the capacitance (C), the surface charge density (σ_b) and the free charge density (σ_f) at O and P points.



1. (15%)A burst of π^+ mesons travels down an evacuated beam tube at Fermi lab moving at 0.92C with respect to the laboratory.
   (a) Compute γ for this group of pions.
   (b) The proper mean lifetime of pions is 2.6×10^{-8}S. What mean lifetime is measured in the lab?
   (c) If the burst contained 50,000 pions, how many remain after the group has traveled 50m down the beam tube?
2. (15%)Suppose a 0.511MeV photon from a positron-electron annihilation scatters at 110° from a free electron.
   (a) What are the energies of the scattered photon and the recoiling electron?
   (b) Relative to the initial direction of the 0.511MeV photon, what is the direction of the recoiling electron?
3. (15%)In the Davisson-Germer experiment, at what angle Φ would the reflected electron beam appear if the accelerating voltage is 102.2 volt and the lattice spacing is 0.24266nm.
4. (20%)The first excited state of the simple harmonic oscillator with the mass m and the potential energy 1/2 kx^2 has a wavefunction of the form $\phi(x) = Axe^{-ax^2}$.
   (a) Find the value a and the energy E.
   (b) Find the constant A from the normalization condition.
   [Formula: $\int_0^\infty e^{-u^2}u^2du = \frac{\sqrt{\pi}}{4}$]
5. (20%)A hydrogen atom is in the ground state. The wavefunction is given by $\Psi_{1,0,0}(r,\theta,\phi)=R_{10}(r)Y_{0,0}(\theta,\phi)$,
   where $R_{10}(r) = 2\,a_0^{-3/2}\exp(-r/a_0)$ and $Y_{0,0}(\theta,\phi)= 1/\sqrt{4\pi}$.
   (a) Find the most probable value of r.
   (b) Find the probability of being found inside the Bohr radius.
   (c) Find the uncertainty value of r.
   [Formula: $\int_0^\infty e^{-x}x^{n-1}dx = (n-1)!$ for integer n]
6. (15%)The transitions of familiar sodium yellow doublet are $3p(^2P_{1/2})\rightarrow 3s(^2S_{1/2})$ λ=589.6nm and $3p(^2P_{3/2})\rightarrow 3s(^2S_{1/2})$ λ=589.0nm.
   (a) Estimate the energy splitting between the $^2P_{3/2}$ and $^2P_{1/2}$ states due to the spin-orbit effect.
   (b) Estimate the magnetic field that 3p electron in sodium experiences.
   Assume $\vec{B}$ is parallel to the Z axis.