Show the details of your work.

1. (10%) The magnetic induction $\vec{B}$ is defined by the Lorentz force equation $\vec{F} = q(\vec{v} \times \vec{B})$. Carrying out three experiments on charge q, we find that if velocity $\vec{v} = \hat{x}$, $\frac{\vec{F}}{q} = -3\hat{z} - \hat{y}$; if $\vec{v} = \hat{y}$, $\frac{\vec{F}}{q} = -2\hat{z} + \hat{x}$; and if $\vec{v} = \hat{z}$, $\frac{\vec{F}}{q} = 2\hat{y} + 3\hat{x}$. From the results of these three separate experiments, calculate the magnetic induction $\vec{B}$.

2. (10 points) Find the eigenvalues and the corresponding orthonormal eigenvectors of the matrix

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

3. (10 points) Find a general solution of nonhomogeneous Euler-Cauchy equation by variation of parameters.

$$x^2y'' - 4xy' + 6y = 3x^4\sin x.$$

4. (15 points) Tank T_1 contains initially 200 gal of pure water. Tank T_2 contains initially 100 gal of water in which 150 lb of fertilizer are dissolved. Liquid circulates through the tanks at a constant rate of 2 gal/min, and the mixture is kept uniform by stirring. Find the amounts of fertilizer $y_1(t)$ and $y_2(t)$ in T_1 and T_2, respectively, where t is time. Solve the eigenvalue problem of a system of two first-order differential equations.

5. (15 points) Solve the initial value problem of a damped mass-spring system by Laplace transform

$$y'' + 2y' + 2y = r(t),\ r(t) = 5\sin 2t \text{ if } 0 < t < \pi \text{ and } 0 \text{ if } t > \pi;\ y(0) = 1,\ y'(0) = -5.$$

6. (15 points) Find the Fourier transform of xe^{-2x^2} without using transform formula table.

7. (15 points) Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ and find the temperature u(x, t) in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $u(x,0) = \sin 0.1\pi x$.

8. (10 points) Use the residue theorem to evaluate the integral $\int_0^{2\pi} \frac{d\theta}{25 - 24\cos\theta}$.

1, (15) Suppose the electric field in some region is found to be $\mathbf{E} = kr^3\hat{\mathbf{r}}$. In spherical coordinates (k is some constant).

(a) Fine the charge density ρ.

(b) Find the total charge contained in a sphere of radius R, centered at the origin.

2, (15) An infinite plane slab, of thickness 2d, carries a uniform volume charge density ρ as shown in Fig. 1. Find the electric field, as a function of y, where y = 0 at the center.

3, (20) Two infinite grounded metal plates lie parallel to the xz-plane, one at y = 0, the other at y = a (Fig.2). The left end, at x = 0, is closed off with an infinite strip insulated from the two plates and maintained at a specific potential $V_0(y)$. Find the potential inside this "slot."

4. (15) A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization

$$\mathbf{P}(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}},$$

where k is a constant and r is the distance from the center (Fig.3). (There is no free charge in the problem.) Find the electric field in all three regions.

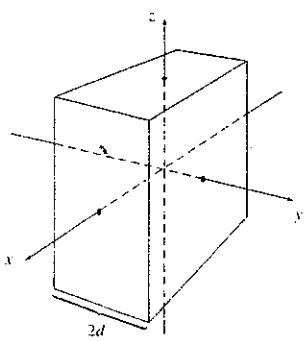

Fig. 1

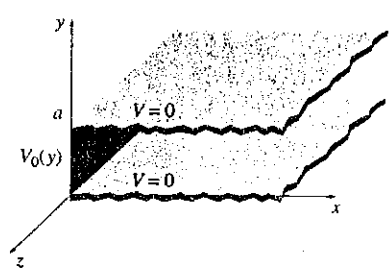

Fig. 2

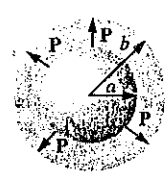

Fig. 3

5. (20) Find the vector potential a distance s from an infinite straight wire carrying a current I and then from the result of vector potential to determine the magnetic field **B**.

6. (15) A square loop, side a, resistance R, lies a distance s from an infinite straight wire that carries current I (Fig. 4). Now turn the current down gradually, so that I drops to zero:

$$I(t) = (1-\alpha t)I \quad \text{for } 0 \le t \le 1/\alpha$$
$$I(t) = 0 \quad \text{for } t > 1/\alpha$$

In what direction does the induce current in the square loop flow, and what total charge passes a given point in the loop during the time this current flows?

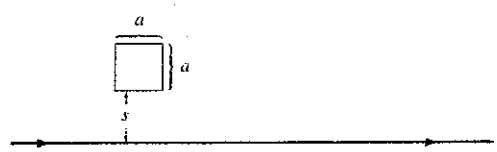

Fig. 4

國立中山大學 九十二學年度 物理系碩士班入學考試 近代物理試題

1. (a) What does a quantized angular momentum vector differ from a classical angular momentum vector? (5%)
   (b) What is the Pauli exclusion principle? (5%)
2. What are the difference between Zeeman splitting and fine structure splitting? (8%)
3. Find the deBroglie wavelength of (a) a 50-GeV electron and (b) An electron moving at $v=10^6 m/s$. (12%)
4. A π meson moving through the laboratory at $v=0.98c$ decay into two gamma rays of equal energies, making equal angles θ with the direction of motion. Find the angle θ and the energies of the two gamma rays. The rest mass of π-on is $m_\pi = 135 MeV$. (10%)
5. An excited state of an atom has a lifetime of 10^{-9} sec. What is the natural line width of the energy level? (7%)
6. Two particles of mass m are attached to the ends of massless rigid rod of length a. The system is free to rotate in three dimensions about the center (but the center point itself is fixed). (a) Show that the allowed energies of this rigid rotor. (7%)
   (b) What is the degeneracy of the nth energy level? (6%)
7. A particle of mass m is in a one-dimensional infinite square well with sides at $x=0$ and $x=a$. Find the eigenstates and eigenfunctions. (8%)
   Suppose we have two non-interacting particles, both of mass m, in this potential.
   (b) Find the wave functions of (i) the ground state and (ii) the first excited state, respectively, and their corresponding system energies if two particles are bosons. (8%)
   (c) Find the wave function of the ground state and the Fermi energy, if two particles are fermions. (8%)
8. Imagine a system in which there are just two linearly independent states:

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

   Suppose the Hamiltonian matrix is $H = \begin{pmatrix} k & g \\ g & k \end{pmatrix}$, where g and h are real constants. The time-dependent Schrödinger equation is $H|\psi\rangle = i\hbar \frac{d}{dt}|\psi\rangle$.

   (a) Find the eigenvalues and normalized eigenvectors of the Hamiltonian. (10%)

   (b) Suppose the system starts out (at $t=0$) in state $|1\rangle$. What is the state at time t? (6%)