國立中山大學九十學年度碩博士班招生考試試題

科目：應用數學【物理系碩士班】

1. A force is described by
$\vec{F} = -\hat{i}\, y/(x^2+y^2) + \hat{j}\, x/(x^2+y^2)$.
(a) Express $\vec{F}$ in circular cylindrical coordinates. (5)
(b) Is $\vec{F}$ a conservative force? why? (5)
(c) Calculate the work done by $\vec{F}$ in encircling the unit circle once counterclockwise. (5)
** operating entirely in circular cylindrical coordinates for (b) and (c)

2. Consider a matrix A
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
(a) Is A a Hermitian matrix? why? (3)
(b) Find its eigenvalues $\lambda_1, \lambda_2, \lambda_3$ for matrix A. (9)
(c) Find the orthonormal eigenvectors corresponding to the eigenvalues in (b).(9)
(d) Construct a transformation matrix R such that
$$R^+AR = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$ is a diagonal matrix. (4)

3. (a) Given $y = x$ is a solution of
$(x^2 + 1)\, y'' - 2\, x\, y' + 2\, y = 0$, where $y' = dy/dx$,
find a linear independent solution by reducing the order. (10)
(b) Find the general solution of $y'' + y = x \sin x$. (10)

4. Evaluate the following intergrations
(a) $\int_0^\infty \frac{dx}{x^4+1}$ (b) $\int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x} dx, \quad 0 < a < 1.$
(20)

5. (a) Expand $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$ in a Fourier series. (15)

(b) Using (a) to evaluate
$$\frac{1}{1\cdot 3} + \frac{1}{5\cdot 7} + \frac{1}{9\cdot 11} + \frac{1}{13\cdot 15} + \cdots = ?$$ (5)

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科目：電磁學【物理系碩士班】

1.A lucite sheet ($\varepsilon_r = 3.2$) is introduced perpendicularly in a uniform electric field $\vec{E}_0 = \vec{a}_x E_0$ in free space. Determine the electric field intensity $\vec{E}_l$, the electric displacement $\vec{D}_l$, and the polarization $\vec{P}_l$ inside the lucite. (10%)

2.Consider a very long coaxial cable. The inner conductor has a radius a and is maintained at a potential V_0. The outer conductor has an inner radius b and is grounded. Determine the potential distribution in the space between the conductors. (15%)

3.An uncharged conducting sphere of radius b is placed in an initially uniform electric field $\vec{E}_0 = \vec{a}_z E_0$. Determine (a) the potential distribution V(R, θ), and (b) the electric field intensity E(R, θ) after the introduction of the sphere. (20%)
(Hint: The Legendre Polynomials $P_n(\cos\theta)$; $P_0(\cos\theta)=1$, $P_1(\cos\theta)=\cos\theta$)

4.Determine the magnetic flux density on the axis of a uniformly magnetized circular cylinder of a magnetic material. The cylinder has a radius b, length L, and axial magnetization $\vec{M} = \vec{a}_z M_0$. (15%)

5.Two magnetic media with permeabilities μ_1 and μ_2 have a common boundary, as shown in Fig.1. The magnetic field intensity in medium 1 at the point P_1 has a magnitude H_1 and makes an angle α_1 with the normal. Determine the magnitude and the direction of the magnetic field intensity at the point P_2 in the medium 2. (10%)

6.An alternating emf has a voltage amplitude of 100 V and frequency of 60 cycle/s. It is connected in series with a resistor of 1 Ω, a self-inductor of 0.003 H and a capacitor of 0.002 F. Determine (a) the amplitude and phase of the current and (b)the potential difference across the resistor, the capacitor and the inductor. (c) Make a phasor diagram. (15%)

7.Assume that N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability μ. The core has a mean radius r_0, a circular cross section of radius a($a \ll r_0$), and a narrow air gap of length ℓ_g, as shown in Fig.2. A steady current I_0 flows in the wire. Determine (a) the magnetic flux density $\vec{B}_f$ in the ferromagnetic core; (b) the magnetic field intensity $\vec{H}_f$ in the core; and (c) the magnetic field intensity $\vec{H}_g$ in the air gap. (15%)

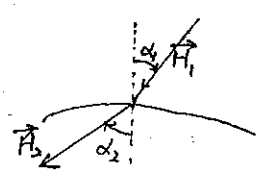

Fig.1

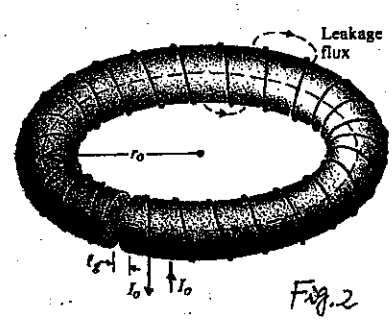

Fig.2

國立中山大學九十學年度碩博士班招生考試試題

科目：近代物理【物理系碩士班】

1. An x-ray photon of 0.0500nm wavelength strikes a free, stationary electron. A photon scatters at 90°. Determine the momenta of the incident photon, the scattered photon, and the electron. (15%)

2. For a particle in the ground state of an infinite one-dimensional well, find (a) Δx, (b)ΔP, and (c)check your result with the uncertainty principle.(20%)

3. For a simple harmonic oscillator of spring constant κ and mass m, one solution of the Schrödinger equation is of the form $\varphi(x) = Ae^{-ax^2}$, a gaussian centered at the origin. Determine fully the wave function and energy in this state. ($\int_{-\infty}^{+\infty} e^{-2ax^2} dx = \sqrt{\frac{\pi}{2a}}$)(15%)

4. The electron of hydrogen atom is the 2P state. (20%)

   $(R_{21}(r) = (\frac{1}{2a_0})^{3/2} \cdot \frac{r}{\sqrt{3}a_0} e^{-\frac{r_0}{2a}})$

   Find (a) the most probable value of r.

   (b) the probability of being found inside the Bohr radius.

5. The fine structure splitting of the $2P_{3/2}$ and $2P_{1/2}$ levels in hydrogen is 4.5×10^{-5}eV. From this, estimate the magnetic field that the 2p electron in hydrogen experiences. Assume $\vec{B}$ is parallel to the z axis. (10%)

6. The first excited state E_2 of the hydrogen atom is 10.2eV above the ground state E_1. What is the ratio of the number of atoms in the first excited state to the number in the ground state at T=5800K? (10%)

7. The energy difference ΔE between the $\ell = 0$ and $\ell = 1$ rotational levels in the CO molecule is found experimentally from measurement of the wavelength $\lambda = 2.6$nm of the corresponding transition. For CO, ΔE is equal to 4.77×10^{-4}eV. Find the equilibrium separation, or bond length r_0, of the CO molecule. (10%)