國立中山大學八十八學年度碩博士班招生考試試題

科 目：應用數學 (物理學系碩士班)　　共 2 頁 第 1 頁

1. A force field is described by $\vec{F} = -\hat{i}\frac{y}{x^2+y^2} + \hat{j}\frac{x}{x^2+y^2}$.

(a) Express $\vec{F}$ in circular cylindrical coordinate. (5%)

(b) Is $\vec{F}$ a conservative force? Why? (5%)

(c) Calculate the work done by $\vec{F}$ in encircling the unit circle centered at the origin once counterclockwise. (5%)

2. Given a matrix $A = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}$

(a) Show that A is a Hermitian matrix. (5%)

(b) find the eigenvalues and the corresponding eigenvectors (normalized), respectively. (10%)

(c) find a transformation matrix R, such that

R^+AR = a diagonal matrix, where $R^+ = (R^*)^t$ = self adjoint matrix of R. (5%)

3. Given $y = x$ is a solution of

$(x^2+1)y'' - 2xy' + 2y = 0$, $y' = \frac{dy}{dx}$

to find the general solution of $y(x)$. (10%)

4. Find the general solution of

$y'' + y = x\sin x$. (10%)

5. Solve the differential equation

$y'' + y = e^{-2t}\sin t$, $y' = \frac{dy}{dt}$

where $y(0) = 0$ & initial conditions
$y'(0) = 0$ (10%)

using Laplace Transform.

6. (a) Expand $f(x)=\begin{cases} 0 & -\pi<x<0 \\ \sin x & 0<x<\pi \end{cases}$ in a Fourier series. (10%)

(b) Using (a) to evaluate

$$\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+\frac{1}{13\cdot 15}+\frac{1}{17\cdot 19}+\cdots\cdots$$ (5%)

7. Evaluate the following integrations (20%)

(a) $\int_0^\infty \frac{dx}{x^4+1}$ (b) $\int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x}$, where $0<a<1$

國立中山大學八十八學年度碩博士班招生考試試題

科　目：電磁學　物理學系碩士班

本試題分為兩部份，第一部份為選擇題每題七分，第二部份為計算問答題

1. 下列敘述何者為正確？
   (a) 在靜電場下必須滿足 $\nabla \cdot \mathbf{D} = \rho$ 及 $\nabla \times \mathbf{E} = 0$
   (b) 在靜磁場下必須滿足 $\nabla \cdot \mathbf{B} = 0$ 及 $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$
   (c) $\nabla \bullet j = -\frac{\partial \rho}{\partial t}$ 所代表的就是電荷守恆定律
   (d) 以上皆是

2. 半徑為 a 與 $b(a<b)$的兩同心導體球殼各帶–2Q 與+Q 的電量如下圖所示，若以一導線連接兩球殼則下列何者為正確
   (a) +Q 的電量會與-2Q 的電量中和後，只剩-Q 的電量於半徑為 a 的球殼
   (b) +Q 的電量會與-2Q 的電量中和後，只剩-Q 的電量於半徑為 b 的球殼以達到導體內部電場為零
   (c) +Q 的電量會與-2Q 的電量中和後，電量重新分配使 $-aQ/(a+b)$的電量分佈於半徑為 a 的球殼，$-bQ/(a+b)$ 的電量分佈於半徑為 b 的球殼以使兩球殼的電位相等
   (d) +Q 的電量會與-2Q 的電量中和後，電量重新分配使 $-a^2Q/(a^2+b^2)$ 的電量分佈於半徑為 a 的球殼，$-b^2Q/(a^2+b^2)$的電量分佈於半徑為 b 的球殼以使兩球殼的電位相等

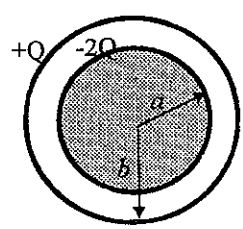


圖 二

3. 一無限大的接地金屬導體平面，在距離其表面為 d 的位值放置一帶電量為 Q 的點電荷，則下列何者為錯誤的敘述?
   (a) 該點電荷受力為 $-\frac{Q^2}{4\pi\varepsilon_0(2d)^2}\hat{z}$
   (b) 該導體表面感應總電荷為-Q
   (c) 導體表面的面電荷密度最大值為 $-\frac{Q}{2\pi d^2}$
   (d) 該系統的靜電位能為 $-\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2d)}$

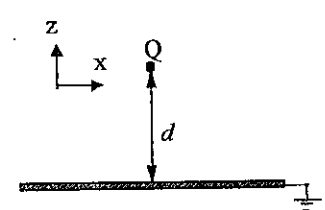


圖 三

4. 一電流源 I 加於半徑為 R 的圓形平行板電容器兩端，如圖四所示，且兩平行板間的距離為 s（假設 s<<R）。當電容器在充電的過程中，請問下列敘述何者為錯誤的？

(a) 有位移電流 I 存在於電容器兩平行板間

(b) 在電容器中有時變電場 $\frac{\partial E}{\partial t}=\frac{I}{\varepsilon_0 A}$ 存在

(c) 在電容器兩平行板間距離中心軸 r (r<R)處有大小為 $B=\frac{\mu_0 Ir}{2\pi R^2}$ 的磁場存在

(d) 在電容器中的電流是因磁場隨時間變化感應產生的

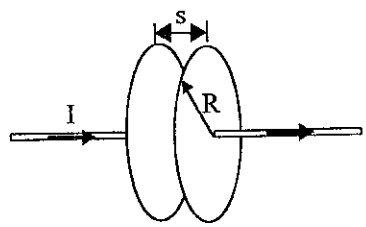


圖 四

5. 一電偶極 $\mathbf{P}=p\hat{z}$，當徑向距離 r 遠大於兩正負電荷所分開的距離時，在球座標為(r,θ,ϕ)位置的電場為

(a) $\mathbf{E}=\frac{p}{4\pi\varepsilon_0 r^3}(\hat{r}2\cos\theta+\hat{\theta}\sin\theta)$

(b) $\mathbf{E}=\frac{p}{4\pi\varepsilon_0 r^2}(\hat{r}2\cos\theta+\hat{\theta}\sin\theta)$

(c) $\mathbf{E}=\frac{p}{4\pi\varepsilon_0 r^3}(\hat{r}\cos\theta+\hat{\theta}2\sin\theta)$

(d) $\mathbf{E}=\frac{p}{4\pi\varepsilon_0 r^2}(\hat{r}2\cos\theta+\hat{\theta}\sin\theta)$

6. 一圓形的電流環帶電流為 I，在一均勻的磁場 $\mathbf{B}=B\hat{k}$ 內，則

(a) 若電流環放置在 x-y 平面上，則其所受的合力為零，合力矩不為零

(b) 若電流環放置在 y-z 平面上，則其所受的合力矩為零但合力不為零

(c) 若電流環放置在 x-z 平面上，則其所受的合力矩為零但合力不為零

(d) 若電流環放置在 x=y 平面上，則其所受的合力矩不為零但合力為零

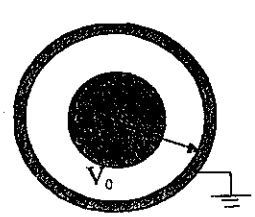
7. 一無限長的同軸電纜，其內徑為 a 且保持 V_0 的電位；外徑為 b 且接地，如下圖所示。

(a) 該同軸電纜每單位長度具有的 $2\pi\varepsilon_0\ln(b/a)$ 電容值

(b) 當 $a\le r\le b$，該同軸電纜的電位分佈為 $V(r)=\frac{V_0}{\ln(b/a)}\ln(b/r)$

(c) 該同軸電纜每單位長度具有的 $\frac{\mu_0}{2\pi\ln(b/a)}$ 電感值

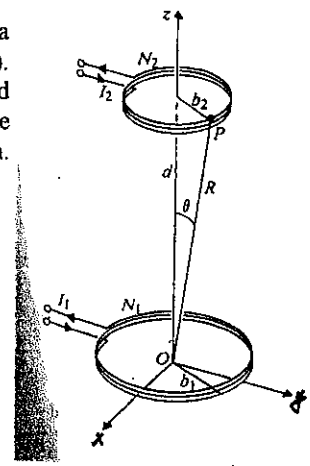
(d) 將該同軸電纜中間部份充塞電介質則每單位長度具有的電容值將減小



第二部份計算問答題共伍拾壹分

1. Find the energy of a uniformly charged spherical shell of total charge q and radius R. (10%)

2. Two coaxial coils of radii b_1 and b_2 are separated by a distance d that is much larger than the radii ($d>>b_1$, b_2). The coils consist of N_1 and N_2 closely wound turns and carry currents I_1 and I_2 respectively. (a) Determine the mutual inductance. (b) Determine the force between them. (16%)



3. An emf V is applied across a parallel-plat capacitor of area S. The space between the conducting plates is filled with two different lossy dielectrics of thinknesses d_1 and d_2, permittivities ε_1 and ε_2, and conductivities σ_1 and σ_2, respectively. Neglecting fringing effect at the edges of the plates, (a) determine the electric field intensities in both dielectrics, and (b) the surface charge densities on the plates and at the interface. (15%)

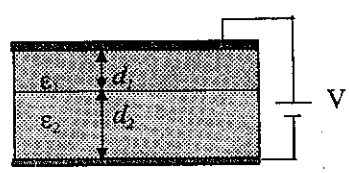


Fig. 3

4. (a) What is Hall coefficient? (b) What information can you get from the Hall coefficient? (10%)

國立中山大學八十八學年度碩博士班招生考試試題

科　目：近代物理 (物理學系碩士班)

1. (1) At what speed does the kinetic energy of a particle equal to its rest energy? (5%)

(2) Two twins of rest mass 60.0kg are headed toward each other in spacecraft whose speeds relative to the earth are 0.800c. What mass does each twin find for the other? (5%)

2.An electron has a de Broglie wavelength of 3.00pm(1pm$=10^{-12}$ m). Find the kinetic energy, the phase velocity and the group velocity. (15%)

3. (1) A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus. (2) A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. (15%)

4.Electron beams with energies of 1.0 eV are incident on a barrier 3.0 eV high and 0.50nm (1nm$=10^{-9}$ m) wide. Find the transmission probability of the electron beams. (15%)

5.Find (1) the most probable value of r for 1s electron, (2) the expectation value of r for 1s electron. ($\psi_{100}(r,\theta,\phi)=\frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-r/a_0}$) (15%)

6.Find the minimum magnetic field needed for the Zeeman effect to be observed in a spectral line of 400 nm wavelength when a spectrometer whose resolution 0.010 nm is used. (10%)

7. The term symbol of the first excited state is $3^2P_{1/2}$. List the possible quantum numbers n, l, j, and m_j of the outer electron in each case. (5%)

8.Suppose we have a system consist of two non-interacting particles, both of mass m, in the infinite square well (two ends 0 and a). Write out the one-particle nth state eigenfunction and eigenvalue. And if the two particles are (1) distinguishable (2) identical boson (3) identical fermion, then find the system eigenfunction and eigenvalue of the ground state. (15%)