國立中山大學八十七學年度碩博士班招生考試試題

科目:(物理系)應用數學

1. 一維波方程式 $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$，其邊界條件為

$$u(0,t)=0, \quad u(L,t)=0 \quad \text{for all } t$$

(15%) 初始條件為

$$u(x,0)=f(x), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$$

求解（以 Fourier series 表之）

2. 求 (a) $\vec{\nabla}\cdot\vec{r} = ?$

(15%)

(b) $\vec{\nabla}\cdot[\hat{r} f(r)] = ?$

(c) $\vec{\nabla}\cdot[\hat{r}\, r^{n-1}] = ?$（應用(b)之結果求取）

(d) $\vec{\nabla}^2(x^2+4y^2+9z^2) = ?$

(e) $\vec{\nabla}\times\left[(x^2+y^2+z^2)^{-\frac{3}{2}}(x\hat{i}+y\hat{j}+z\hat{k})\right] = ?$

3. 求 eigenvalues 和 eigenvectors

(a) (8%) $\begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}$

(b) (8%) $\begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix}$

4. 解微分方程

(a) $(2xe^{2xy}+\cos y)y' + 2ye^{2xy} = 0$ (8%)

(b) $y''+6y'+9y = 18\cos 3x$ (8%)

5. 以 Laplace transformation 方法解微分方程

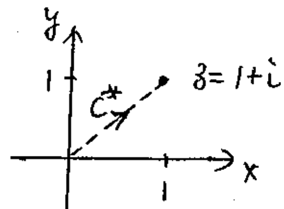
(a) $y''-2y'+y = e^t + t$ with $y(0)=1$, $y'(0)=0$ (8%)

(b) $y''+2y = r(t)$ with $y(0)=0$, $y'(0)=0$

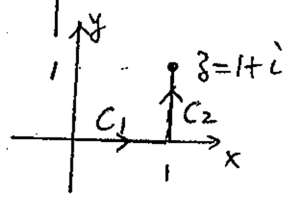
$$r(t) = \begin{cases} 1 & \text{if } 0<t<1 \\ 0 & \text{otherwise} \end{cases}$$ (8%)

6. 積分

(a) (4%) $\int_{C^*} \mathrm{Re}\, z \, dz = ?$



(b) (4%) $\int_{C} \mathrm{Re}\, z \, dz = ?$ $C = C_1 + C_2$



(c) (6%) $\oint_C \frac{7z-6}{z^2-2z} dz = ?$ (C: unit circle) counterclockwise

(d) (4%) $\int_{-\infty}^{\infty} e^{-ax^2} dx = ?$

(e) (4%) $\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = ?$

國立中山大學八十七學年度碩博士班招生考試試題

科 目：電磁學(物理所)

1. Write down the **E** and **B** fields immediately outside the surface of a perfect conductor. Explain. (5%)

2. Write the expressions for time-harmonic retarded scalar and vector potentials in terms of charge and current distributions. (6%)

3. What is the dispersion of a signal? (5%)

4. Write the boundary conditions of **B** and **H** at the interface of a free space and a magnetic material with an infinitely large permeability. (6%)

5. Write the integral and differential forms of Maxwell's equations in MKS unit system. Are all Maxwell's equations independent? Explain. (20%)

6. A cylindrical capacitor of length L consists of coaxial conducting surfaces of radii a and b. The dielectric material between the surfaces has a relative permittivity $\varepsilon_r = 2 + (4/r)$. (a) Determine the capacitance of this capacitor. (b) Find the electrostatic energy stored in the dielectric region. (Neglect the fringing of the electric field at the edge.) (16%)

7. An air coaxial transmission line has a solid inner conductor of radius a and a very thin outer conductor of inner radius b. (a) Determine the magnetic flux density in all space.(6%) (b) Determine the inductance per unit length of the line. (5%) (c) How much magnetic energy per unit length is stored in the system? (5%)

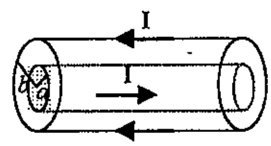


Fig.1

8. The E-field of a uniform plane wave propagating in a dielectric medium is given by
$$E(t,z) = \hat{i}3\cos(10^8 t - z/\sqrt{3}) - \hat{j}\sin(10^8 t - z/\sqrt{3}) \quad \text{(V/m)}$$

(a) Determine the frequency and wavelength of the wave.
(b) What is the dielectric constant of the medium?
(c) Describe the polarization of the wave.
(d) Find the corresponding **H**-field. (16%)

9. If the constant electric field in Fig. 2 has a magnitude E_0, calculate the total electric flux through the parabolodial surface S. (10%)

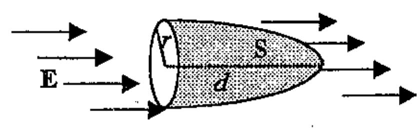


Fig. 2

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科 目：近代物理（物理所）

I.塡充題(每格 4 分, 共 40 分)(Boltzmann's constant k = 1.38×10^{-23} J/K, Planck's constant h = 6.626×10^{-34} J/s, Rydberg constant R = $1.097\times10^{7}m^{-1}$)

1.A stationary body explodes into two fragments with equal rest mass of 1.0 kg and moving apart at a speed of 0.6c relative to the original body. The rest mass of the original body is __(1)__ kg.

2.A spacecraft's antenna is oriented at an angle of 10^0 relative to the axis of the spacecraft. If the spacecraft moves away from the earth at a speed of 0.7c, the angle of the antenna as seen from the earth is __(2)__.

3.The shortest wavelength presents in the radiation from an x-ray machine is __(3)__ (whose accelerating potential is 50,000 V).

4.X-rays of wavelength 10.0 pm(1pm = 10^{-12}m) are scattered from a target. The maximum wavelength presents in the scattered x-rays is __(4)__ and the maximum kinetic energy of the recoil electrons is __(5)__.

5.An electron has a de Broglie wavelength of 2.00 pm, the kinetic energy of this electron is __(6)__.The phase velocity is __(7)__ and the group velocity is __(8)__.

6.A measurement determines the position of a proton with an accuracy of $\pm1.00\times10^{-11}$m. Assume v<<c, the uncertainty in the proton's position 1.00 s later is __(9)__.

7.The wavelength of the H_α presents in the Balmer series of hydrogen is __(10)__.

II.計算題(共 60 分)

1.Consider a beam of charged particles (with charge q) with a kinetic energy of E moving along the x-axis of a system of two electrodes which are held at a voltage difference of V_0 as shown in Fig.1, where $E < V_0 q$. (1) Find the eigenfunctions. (2) Find the reflection probability. (3) Find the probability ratio to find the particle at x = 0 and x = 0.005m. (15 分).

2.An electron in the Coulomb field of a proton is in a state described by the wave function; $\frac{1}{6}[4\psi_{100}(\vec{r})+3\psi_{211}(\vec{r})-\psi_{210}(\vec{r})+\sqrt{10}\psi_{21-1}(\vec{r})]$,

(1)What is the probability in each state? (5 分)

(2)What is the expectation value of the energy? (5 分)

(3)What is the expectation value of L^2 and L_z? (5 分)

3.For the carbon atom, find (1)the electronic configurations (2)the possible states (3)its ground state (4)the Lande g factor of the $^3P_1$ state and (5)the energy level splitting of the $^3P_1$ state under a 0.1 T magnetic field. (20 分)

4.Suppose we put a delta-function bump $H' = \alpha\delta(x-a/2)$ (where α is a constant) in the center of the one dimensional infinite square well. Find the first-order correction to the allowed energies. (10 分)

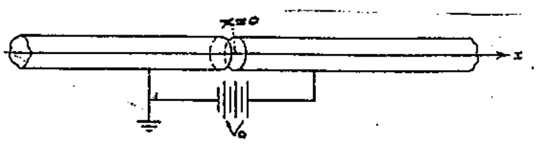

Fig. 1