國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：應用數學【物理系碩士班】

題號：4045

1. Find the radial components of the velocity and acceleration v_ρ and a_ρ of a moving particle in circular cylindrical cooridantes.
Hint: $\mathbf{r}(t)=\hat{\rho}(t)\rho(t)+\hat{z}z(t)=[\hat{x}\cos\varphi(t)+\hat{y}\sin\varphi(t)]\rho(t)+\hat{z}z(t)$,
where $\hat{\rho}$ is the unit vector along the radial direction.(10%)

2. $\delta(g(x))$ is the delta function of a function $g(x)$ with simple zeros at $x=a_i$ in the real axis. Show that

$$\delta(g(x))=\sum_{\substack{a_i,\\ g(a_i)=0,\\ g'(a_i)\neq 0}}\frac{\delta(x-a_i)}{|g'(a_i)|}.(10\%)$$

3. Evaluate the function of matrix $\exp(i\sigma_x\varphi)=?$,
where i is the imaginary unit,
and $\sigma_x=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, one of the Pauli matrices.(10%)

4. A boat, coasting through the water, experiences a resisting force proportional to v^n, v being the boat's instantaneous velocity. Newton's second law leads to

$$m\frac{dv}{dt}=-kv^n.$$

With $v(t=0)=v_0, x(t=0)=0$, find v as a function of time and v as a function of distance.(10%)

5. A quantum mechanical analysis of the Stark effect leads to the differential equation

$$\frac{d}{d\xi}\left(\xi\frac{du}{d\xi}\right)+\left(\frac{1}{2}E\xi+\alpha-\frac{m^2}{4\xi}-\frac{1}{4}F\xi^2\right)u=0.$$

Here α is a separation constant, E is the total energy, and F is a constant. Using the larger root of the indical equation, develop a power-series solution about ξ=0. Evaluate the first three coefficients in terms of zero order coefficient a_0. Note that the constant F does not appear untial a_3 is included. (10%)

6. A triangular wave is represented by

$$f(x)=\begin{cases} x, & 0<x<\pi \\ -x, & -\pi<x<0. \end{cases}$$

Represent $f(x)$ by a Fourier series.(10%)

7. The function $f(x)=\begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$ is a symmetrical finite step function.

Find the Fourier cosine transform of $f(x)$.(10%)

8. (a) Evaluate the integral $\int_{-\infty}^{\infty}\frac{\cos x}{x^2+a^2}dx$.(5%)

(b) Evaluate the integral $\int_{-\infty}^{\infty}\frac{x\sin x}{x^2+a^2}dx$.(5%)

9. An atom particle is confined inside a rectangular box of sides $a, b,$ and c. The particle is described by a wave function Ψ that satisfies the Schrodinger wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi=E\Psi.$$

The wave function is required to vanish at each surface of the box. The condition imposes costraints on the separation constants and therefore on the energy E. What is the smallest value of E for which such a solution can be obtained?(10%)

10. Green's function $G(\mathbf{r}_1,\mathbf{r}_2)$ is a solution of the equation $\nabla^2 G(\mathbf{r}_1,\mathbf{r}_2)=-\delta(\mathbf{r}_1-\mathbf{r}_2)$, where delta function $\delta(\mathbf{r}_1-\mathbf{r}_2)$ describes a point source at the point $\mathbf{r}_2$.

Evaluate the Green's function of three dimensional free space.(10%)

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：普通物理【物理系碩士班】

題號：4046

1. A block of mass M rests on a frictionless horizontal table and is connected to two fixed posts by spring having spring constants k_1 *and* k_2 respectively, as shown in Fig. 1.
   (a) If the block is displaced slightly from its equilibrium position, what is the frequency of vibration? (5%)
   (b) Suppose that the block is vibrating with amplitude A and that, at the instant that it is passing through its equilibrium position, a mass *m* is dropped vertically onto the block and sticks to it. Find the new frequency and new amplitude of vibration. (10%)

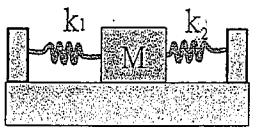


Fig. 1

2. A marble of mass M and radius *R* rolls without slipping on an inclined plane making an angle θ with the horizontal.
   (a) Calculate the acceleration of the center of the marble. (8%)
   (b) If the marble is started with initial velocity v_0 directly up the inclined plane, how long will it be before the marble returns to its starting point? (7%)

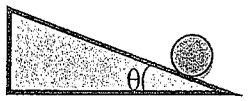


Fig. 2

3. A long coaxial cable consists of two concentric cylindrical conductors of radii *a* and *b* and length *h* as shown in Fig. 3. The inner conductor is assumed to be a thin cylindrical shell. The conductors carry current *I* in opposite directions.
   (a) Calculate the self-inductance *L* of this cable. (8%)
   (b) Calculation the total energy stored in the magnetic field of the cable. (7%)

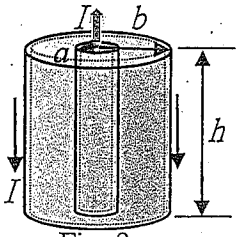


Fig. 3

4. A *Faraday disk* consists of a rotating conducting disk with one stationary brush (a sliding electrical contact) at its axle, A, and another at a point, B on its circumference, as shown in Fig. 4.
   (a) A magnetic field $\vec{B}$ is applied perpendicular to the plane of the disk which the angular speed is ω, and the radius of the disk is *R*. Find the *emf* generated between the brushes. (8%)
   (b) If a resistor was connected between A and B, what is the direction of current passed through the resistor? (7%)

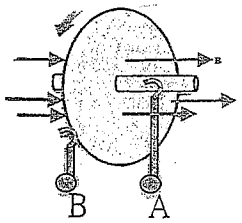


Fig. 4

5. An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown in Fig. 5. Calculate that the net work done on the gas in the entire cycle. (15%)

6. The hydrogen spectrum has a red line at 656*nm* and a blue line at 434*nm*. What are the angular separations between two spectral lines in first order obtained with a diffraction grating that has 4500 grooves/*cm*? (10%)

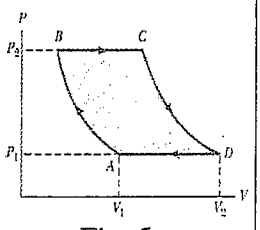


Fig. 5

(請注意，下面尚有題目)

7. As shown in Fig. 6, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod (not a string) of length and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle? (15%)

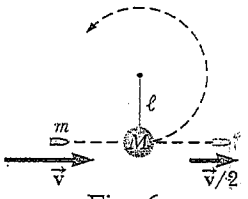


Fig. 6

中山大學 101 學年度碩士暨碩士專班招生考試試題

題號：4047

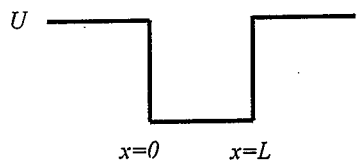
科目：近代物理【物理系碩士班】

1. (15%) A bus is moving in the positive x direction at speed 0.5c with respect to the ground. An observer in a car is driving at speed 0.8c in a direction 60 degrees from the positive x axis (relative to the ground). A passenger on the bus travels for one hour according to his own watch. How long (in the unit of hour) is this trip according to an observer in the car? c is the speed of light.

2. (20%) The wavefunction
$$\Psi(x) = Cxe^{-\alpha x^2}$$
also describes a state of the quantum oscillator, provided the constant α is chosen appropriately.
(a) (7%) Using the Schrödinger's equation, obtain an expression for α in terms of the oscillator mass m and the classical frequency of vibration ω.
(b) (7%) What is the energy of this state?
(c) (6%) Find <x>. Note: $\int x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \quad a > 0$

3. (20%) Consider the scattering of particles from the potential well shown in the right figure. Note that the potential $U(x) = U$ for $x < 0$ and $x > L$, and $U(x) = 0$ for $L \geq x \geq 0$. $E > U$ in all regions. Assuming that the particle is incident from the left. What is the condition for no reflection in the region $x < 0$? Please explain it and prove it using the Schrödinger equation.

4. (15%) A particle with the orbital angular moment $L=2\ \hbar$ and spin $S=1\ \hbar$. If the spin-orbital interaction Hamiltonian is of the form
$H_{so} = A\vec{L}\cdot\vec{S}$.
Find the eigenenergies and degenera
constant.



5. (15%) Show that when a photon of energy E is scattered from a free electron at rest with mass m_e, the maximum kinetic energy of the recoiling electron is given by
$$K_{\max} = \frac{E^2}{E + \frac{1}{2}m_e c^2}$$

6. (15%) Explain (a) (6%) the Bohr's quantum model of the atom, (b) (6%) the Franck-Hertz experiment, and (c) (3%) the relation between (a) and (b).